UNCW HIGH SCHOOL MATH CONTEST

Department of Mathematics and Statistics

April 2, 2004

Scoring

Each question answered correctly scores 4 points.
Each question left blank scores 1 point.
Each questions answered incorrectly scores 0 points.

2004 High School Mathematics Contest

1. In a math class grades are computed with homework worth 25%, two midterms worth 25% each and a final exam worth 25%. If 19 homework assignments of equal value were graded, what percent of a student's grade is one homework assignment?

   A. 1.3% B. .76% C. .52% D. .013% E. 5.26%

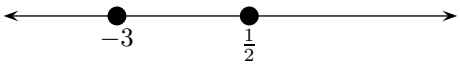
2. Compute $\frac{2m^2-5m-12}{m^2-10m+24} \div \frac{4m^2-9}{m^2-9m+18}$.

   A. $\frac{m^2+m-12}{2m^2-5m-12}$ B. $\frac{m-3}{2m+3}$ C. $\frac{1}{2}$ D. $\frac{m-3}{2m-3}$ E. $\frac{m^2-9m+18}{2m^2-11m+12}$

3. At a party, everyone shook hands with each other exactly once. How many people attended the party if there were 78 handshakes?

   A. 13 B. 29 C. 39 D. 77 E. 79

4. Determine which graph is the solution to the inequality $2y^2 + 5y > 3$.

   A.
   

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   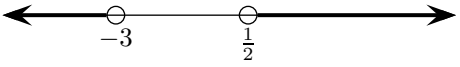

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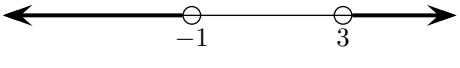
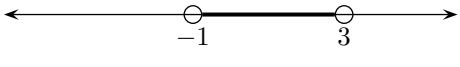
   

   D.
   

   E.
   

5. Factor $(4x^2+1)^2(2x-1)^{-1/2}+16x(4x^2+1)(2x-1)^{1/2}$.

A. $\frac{(4x^2+1)(36x^2-16x+1)}{\sqrt{2x-1}}$

B. $(4x^2+1)(2x-1)^{1/2}(8x^3+4x^2+18x+1)$

C. $\frac{2(2x^2+1)(10x^2-8x+1)}{\sqrt{x-1}}$

D. $\frac{(4x^2+1)(4x^2+16x+1)}{\sqrt{2x-1}}$

E. $\frac{(4x^2+1)^2(18x^2-16x+1)}{\sqrt{2x-1}}$

6. For $f(x)=\frac{1}{x}$, determine $\frac{f(x+h)-f(x)}{h}$.

A. 1 B. $\frac{1}{h^2}$ C. $\frac{1}{h(x+h)}$ D. $\frac{-1}{x(x+h)}$ E. $\frac{1}{xh}$

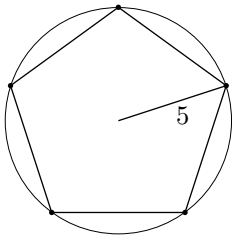
7. Determine which of the following numbers become smaller when squared.

A. $-\frac{1}{2}$ B. 0 C. $\frac{6}{7}$ D. $\frac{9}{8}$ E. 1.001

8. A sequence of numbers is defined recursively by $a_1=1, a_2=1$, and for $k>2$, $a_k=\frac{1}{4}a_{k-2}+\frac{1}{3}a_{k-1}$. Evaluate the infinite sum $a_1+a_2+a_3+\cdots$.

A. 3 B. 4 C. 8 D. 12 E. 20

9. Find the perimeter, to the nearest centimeter, of a regular pentagon inscribed in a circle of radius 5 cm.



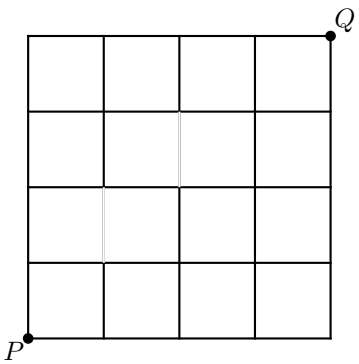
A. 23 B. 25 C. 27 D. 29 E. 31

10. On the calendar page shown, a 3×3 block of numbers has a sum of 198. Determine the smallest number in the block.

Sun	Mon	Tue	Wed	Thu	Fri	Sat
				1	2	3
4	5	6	7	8	9	10
11	12	13	14	15	16	17
18	19	20	21	22	23	24
25	26	27	28	29	30	

A. 6 B. 9 C. 10 D. 13 E. 14

11. Find the number of paths from P to Q taking steps only Northward or Eastward at each intersection.



A. 24 B. 28 C. 32 D. 34 E. 38

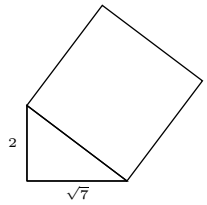
12. A function $f(n)$ defined for all positive integers has the property that $f(m) + f(n) = f(mn)$. If $f(2) = 7$ and $f(3) = 10$, calculate $f(36)$.

A. 24 B. 27 C. 34 D. 149 E. 4900

13. Which of the following numbers is the largest?

A. 10000^{100} B. 2^{10000} C. 1000^{1000} D. 5^{4000} E. 3^{5000}

14. A square is constructed on the hypotenuse of a right triangle whose legs have lengths 2 and $\sqrt{7}$, forming a pentagon. Determine the area of this pentagon.



A. $3+\sqrt{7}$ B. $5+\sqrt{7}$ C. $9+\sqrt{7}$ D. $11+\sqrt{7}$ E. $13+\sqrt{7}$

15. Compute 5^A, where $A = \frac{(\log_5 1 - \log_5 4)(\log_5 25 - \log_5 2)}{(\log_5 1 - \log_5 25)(\log_5 8 - \log_5 4)}$.

A. $\frac{25}{2}$ B. 2 C. 5 D. 25 E. $\log_5 25$

16. If $a \otimes b = ab - 3a + 1$, determine $5 \otimes (7 \otimes 5)$.

A. 43 B. 61 C. 71 D. 101 E. 151

17. Natasha's daughter is twice as old as Natasha's son and half as old as Natasha. In 22 years, Natasha's son will be half Natasha's age. Determine the age of Natasha's daughter.

A. 11 B. 18 C. 22 D. 32 E. 44

18. Let a, b, c be the distinct complex roots of $x^3 + 3x + 4$. Find $a^3 + b^3 + c^3$.

A. -20 B. -12 C. -4 D. 4 E. 12

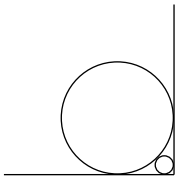
19. Find the angle θ in radians for which $0 \leq \theta \leq \pi$ and $4\cos 2\theta + 4\sin^2\theta - 4\cos\theta = -1$.

A. $\frac{\pi}{6}$ B. $\frac{\pi}{4}$ C. $\frac{\pi}{3}$ D. $\frac{\pi}{2}$ E. $\frac{2\pi}{3}$

20. The product of four positive consecutive integers is 3024. Determine the sum of the integers.

A. 10 B. 30 C. 37 D. 40 E. 47

21. In the figure, the two circles are tangent to the right angle and to one another. If the larger circle has radius 1, determine the radius of the smaller circle.



A. $\sqrt{2}$ B. $\frac{\sqrt{2}-1}{2}$ C. $\frac{\sqrt{2}+1}{2}$ D. $3-2\sqrt{2}$ E. $3+2\sqrt{2}$

22. Suppose that w and z are such that both $(1+2i)w$ and $(1+2i)z$ are different real numbers. What is the slope of the line connecting w and z in the complex plane?

A. -2 B. -1 C. 0 D. 1 E. 2

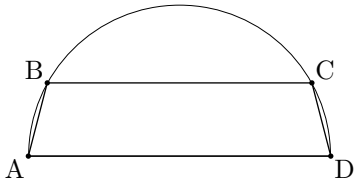
23. Given that $x+3y+7z=100$ and $x+5y+13z=150$, determine $x+y+z$.

A. 40 B. 50 C. 60 D. 70 E. 80

24. Determine the remainder upon division of 4^{2004} by 7.

A. 1 B. 2 C. 3 D. 4 E. 5

25. The isosceles trapezoid $ABCD$ is inscribed in a semicircle such that the trapezoid and the semicircle share the same base, $\overline{AD} = 4$, and $\overline{AB} = \overline{CD} = 1$. Compute $\overline{BC}$.



A. $\frac{7}{4}$ B. 2 C. $\frac{7}{2}$ D. $2\sqrt{2}$ E. 4

26. How many integers x are there such that $1 \leq x \leq 100$ and $x^3 + 3x + 1$ is divisible by 5?

A. 22 B. 28 C. 34 D. 40 E. 46

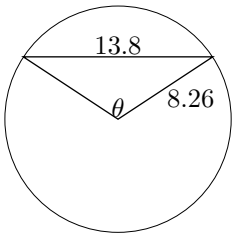
27. For each real number x, let $f(x)$ be the minimum value of the numbers $4x$, x^2, and $-2x+3$. Then the maximum value of $f(x)$ is:

A. 1 B. $\frac{5}{3}$ C. $\frac{8}{3}$ D. $\frac{17}{6}$ E. 3

28. Find the sum of the series $\frac{1}{2} + \frac{1}{6} + \frac{1}{12} + \frac{1}{20} + \frac{1}{30} + \cdots$, whose nth term is $\frac{1}{n^2+n}$.

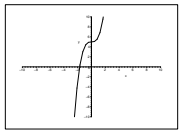
A. ∞ B. $\frac{3}{4}$ C. 1 D. $\frac{4}{3}$ E. $\frac{5}{3}$

29. Find the measure, to the nearest degree, of a central angle subtended by a chord of length 13.8 cm in a circle of radius 8.26 cm.



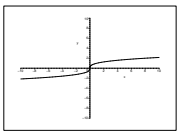
A. 33 B. 45 C. 73 D. 113 E. 140

30. Consider the graph of a function $f(x)$ depicted below.

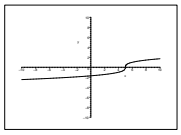


Determine which of the following is the graph of the inverse of $f(x)$.

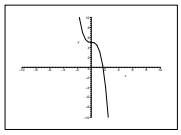
A.



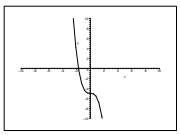
B.



C.



D.



E.