Schedule of the Lectures

Wednesday, December 13

All lectures will be held in room Waaier 4 of building 12 called Waaier.

09.30 – 10.00	Registration, Coffee and Tea
10.00 – 10.05	Welcome and Opening
10.05 – 11.05	**A. Marshakov** Instantons, localisation, partitions and the Toda hierarchy in Seiberg-Witten theory
11.05 – 12.05	**I. Strachan** Duality for Jacobi group orbit spaces and elliptic solutions of the WDVV equations
12.05 – 13.30	Lunch Break
13.30 – 14.30	**S. Loktev** Weyl modules over multi-variable current algebras
14.30 – 15.30	**E. Emsiz** Affine Weyl groups and integrable systems with delta-potentials
15.30 – 15.45	Coffee and Tea
15.45 – 16.45	**A. Kiselev** Pre-Hamiltonian structures for nonlinear integrable systems
16.45 – 17.45	**A. Losev** TBA
18.30	Reception and Dinner

Thursday, December 14

All lectures will be held in room Waaier 4 of building 12 called Waaier.

09.00 – 10.00	**Y. Kajihara** Multiple hypergeometric transformations with different dimensions and related topics
10.00 – 11.00	**N. Jing** Quantum homogeneous spaces and bi-invariants
11.00 – 11.30	Coffee and Tea
11.30 – 12.30	**D. Jakelic** On representations of quantum groups: crystal basis and completions
12.30 – 13.45	Lunch Break
13.45 – 14.45	**T.A. Springer** An extension of Bruhat′s lemma
14.45 – 15.45	**G. Schwarz** Geometric Invariant Theory of Real Groups
15.45 – 16.15	Coffee and Tea
16.15 – 17.15	**A.G. Helminck** On orbits of spherical subgroups acting on symmetric varieties
18.30	Conference Dinner

Friday, December 15

All lectures will be held in room Waaier 4 of building 12 called Waaier.

09.00 – 10.00	**D. Zvonkine** Tautological relations in the cohomology rings of moduli spaces and the r-spin Witten conjecture
10.00 – 11.00	**S. Kharchev** The Toda chain : Integrability and representation theory
11.00 – 11.30	Coffee and Tea
11.30 – 12.30	**K. Slooten** TBA
12.30	Lunch

Abstracts

Instantons, localisation, partitions and the Toda hierarchy in Seiberg-Witten theory

A. Marshakov

We formulate effective Seiberg-Witten theory for the $N = 2$ supersymmetric gauge theory in terms of integrable systems and show how its effective geometry appears in the quasiclassical computation of Nekrasov instanton partition function. Localisation expresses the integrals over instantonic moduli space as a sum over partitions, which is rewritten as a 'Toda-like' matrix element in terms of the fermionic representation of $GL(\infty)$. The quasiclassical computation of this matrix element is performed, and the result is expressed in terms of Krichever tau-function of the quasiclassical hierarchy of Toda type or the generalised Seiberg-Witten prepotential. The integrable Toda hierarchy of the Seiberg-Witten theory is therefore derived directly from the microscopic instanton approach and this justifies many existing hypothesis about the holomorphic sector of nonperturbative $N = 2$ theory.

Duality for Jacobi group orbit spaces and elliptic solutions of the WDVV equations

I. Strachan

From any given Frobenius manifold one may construct a so-called dual structure which, while not satisfying the full axioms of a Frobenius manifold, shares many of its essential features, such as the existence of a prepotential satisfying the WDVV equations of associativity. Jacobi group orbit spaces naturally carry the structures of a Frobenius manifold and hence there exists a dual prepotential. In this paper this dual prepotential is constructed and expressed in terms of the elliptic polylogarithm function of Beilinson and Levin.

Weyl modules over multi-variable current algebras

S. Loktev

This talk is about representations of the Lie algebra of multi-variable polynomials (or, more generally, functons on an affine variety) with values in a reductive Lie algebra.

We introduce Weyl modules as the universal finite-dimensional representations, generated by a highest weight vector (common eigenvector of currents with values in the Borel subalgebra). Then we discuss properties of these modules, relate them to the well-known space of diagonal harmonics and propose some formulas for dimension.

The talk is based on the papers by V.Chari, B.Feigin, A.N.Kirillov and the speaker.

Affine Weyl groups and integrable systems with delta-potentials

E. Emsiz

I will talk about root system generalizations of one-dimensional quantum systems of spin-particles on a ring with pair-wise contact interaction (delta-potential). The symmetry of the quantum system turns out to be controlled by the Cherednik's degenerate double affine Hecke algebra H at level zero. Concretely, we show that the quantum system naturally arises from a basic representation of H in terms of certain vector-valued Dunkl-type operators. Crucial is the fact that H has a large center.

In my talk I will mainly concentrate on root system generalizations of one-dimensional quantum systems of spin-less particles on a ring with repelling pair-wise contact interaction. In this case many aspects of the theory are well-understood. It is known for example that these generalizations are completely integrable, that there exists a root systems version of Pauli's exclusion principle for interacting bosons and that the spectrum is completely discrete. I shall also sketch why the Bethe ansatz eigenfunctions are complete in the space of L^2-integrable functions with respect to a fundamental alcove.

The major part of my talk is joint work (in progress) with Eric Opdam and Jasper Stokman.

Pre-Hamiltonians structures for nonlinear integrable systems

A. Kiselev

We review properties of the pre-Hamiltonian operators whose image is closed with respect to the commutation; this is a generalization of the standard Poisson structures for evolutionary PDE systems. Natural examples of the pre-Hamiltonian operators are given through the factors in higher symmetries of the Liouville-type Euler-Lagrange systems; we give a complete description of the symmetry Lie algebras for these hyperbolic equations. We recall that commutative subalgebras of their Noether's symmetries provide the principal KdV-type hierarchies, the Drinfeld-Sokolov equations on the 2D Toda systems associated with the semi-simple Lie algebras; thus we reveal the pre-Hamiltonian structure of these and similar integrable systems. Independently from the approach through the Liouville-type equations, we show that the non-standard structures that determine the 3-component dispersionless Boussinesq equation (the structures have been found recently by Kersten et al.) are also pre-Hamiltonian. Various applications of the pre-Hamiltonian formalism to deformation theory of PDE and the W-algebras will be discussed.

TBA

A.Losev

Multiple hypergeometric transformations with different dimensions and related topics

Y. Kajihara

About 30 years ago, hypergeometric series in $SU(n+1)$ (or hypergeometric series of type A_n) have been introduced by Holman, Biedenharn and Louck in need of the explicit expressions of the Clebsch-Gordan coefficients for irreducible representation of the unitary group $SU(n+1)$. Including its basic (often called as MilneÕs class), elliptic and root system analogues, multiple hypergeometric series have been investigated in several points of view. Also some applications have been investigated: for example, infinite series of the sum of squares formulae by S.C.Milne and multivariate orthogonal polynomials of Heckman-Opdam type by several authors.

In this talk, I will present some transformation formulae for multiple hypergeometric series of type A with different dimensions by starting from the Cauchy's reproducing kernel. In the course of the derivations, symmetries of the Cauchy kernel and a certain divided difffence operator (a special case of Macdonald's q-difference operators) will be used. By combining the transformation formulae with different dimensions, a number of hypergeometric transformations of type A_n can be obtained including known ones. Symmetries of several classes of hypergeometric series of type A_n will also be discussed including the $n = 1$ case. There, a Coxeter group which has not been in the literature arises as a group descibing the symmetries of a class of A_n hypergeometric series. If time remains, I will present applications of multiple hypergeometric transformations with different dimensions such as partition function identities and the Poisson kernels for some orthogonal polynomials.

References

1. Yasushi Kajihara: Euler transformation formula for multiple basic hypergeometric series of type A and some applications, Advances in Mathematics 187(2004), pp. 53-97.

2. Yasushi Kajihara: On multiple hypergeometric transformation formulae arising from the balanced duality transformation, preprint.

Quantum homogeneous spaces and bi-invariants

N.Jing

Quantum symmetric spaces of quantum linear space $U_q(SU(2n))$ modulo $U_s(Sp(n))$ will be studied. The ring of invariants are classified in terms of generators and relations. We also determine the bi-invariants as certain q-analog of zonal spherical functions and discuss the relations to Macdonald symmetric funtions. Our work enables us to define a q-analog of Phaffians. (joint work with R. Rays)

On representations of quantum groups: crystal basis and completions

Dijana Jakelic

Crystal bases, defined by Kashiwara, provide an extremely powerful tool in the study of certain representations of quantum groups. Completion was introduced by Enright on a category of representations of a complex semisimple Lie algebra as an effective process of obtaining new representations from a given one, the latter sitting as a subrepresentation. I will discuss some results and issues in bringing the concepts of crystal bases and Enright's completions together.

An extension of Bruhat′s lemma

T.A.Springer

The talk is about decompositions of a reductive group, of which the Bruhat decomposition is a particular case. They are related to decompositions in the compactification of an adjoint group, recently used by Lusztig and Xuhua He. The relation will also be discussed.

Geometric Invariant Theory of Real Groups

G. Schwarz

We investigate actions of a real Lie group G on complex spaces. Using moment map techniques we establish the existence of a quotient and a version of Luna's slice theorem as well as a version of the Hilbert-Mumford criterion. In the case of G-actions on real manifolds we generalize the stratification results of Kirwan (which are in the context of reductive complex group actions on Kähler manifolds).

On orbits of spherical subgroups acting on symmetric varieties

Aloysius Helminck

Let G be a reductive linear algebraic group, and H, K spherical subgroups of G. The double cosets $H\backslash G/K$ and some variations have been studied for many types of spherical subgroups, In this talk we discuss some recent results about these double cosets for two types of spherical subgroups,. First for $H = P$ a parabolic subgroup and K a symmetric subgroup we discuss some combinatorial results about the ordered poset related to these double cosets. After that we discuss some results for the case that both H and K are symmetric (and defined over the real numbers).

Tautological relations in the cohomology rings of moduli spaces and the r-spin Witten conjecture

D. Zvonkine

Let $\alpha \in H^*(\overline{\mathcal{M}}_{g,n})$ be a tautological cohomology class of the moduli space of stable curves. In Givental's theory of formal Gromov-Witten potentials it is meaningless to speak of α, but intersection numbers of α with other classes are well-defined. Using a recent work of Y.-P. Lee, we prove that if $\alpha = 0$ then all these intersection numbers vanish for all potentials. We also show that the converse statement is equivalent to Gorenstein's conjecture on the tautological cohomology ring. This result is a first step in the task of establishing an equivalence between formal and geometric Gromov-Witten potentials.

As the most important application we show that our results suffice to deduce the statement of a 1991 Witten conjecture on r-spin structures from the results obtained by Givental on the corresponding formal Gromov-Witten potential.

The conjecture in question states that certain intersection numbers on the moduli space of r-spin structures can be arranged into a power series that satisfies the r-KdV (or rth higher Gelfand-Dikii) hierarchy of partial differential equations. This is joint work with C. Faber and S. Shadrin.

The Toda chain : Integrability and representation theory

S.Kharchev

In the framework of the Quantum Inverse Scattering Method, we express the $GL(N,R)$ Toda wave function in terms of multiple integrals of the Mellin-Barnes type. Using the representation theory of $gl(N)$, the same result is obtained in terms of pairing of two mutually dual Whittaker modules. The main tool is the generalization of the Gelfand-Zetlin method to the case of infinite-dimensional representations of $gl(N)$. The interpretation of this generalized construction in terms of the coadjoint orbits is given and the connection with the Yangian $Y(gl)(N))$ is discussed. Further, we present the generalization of the above construction to the case of an arbitrary semisimple Lie group.

TBA

K.Slooten