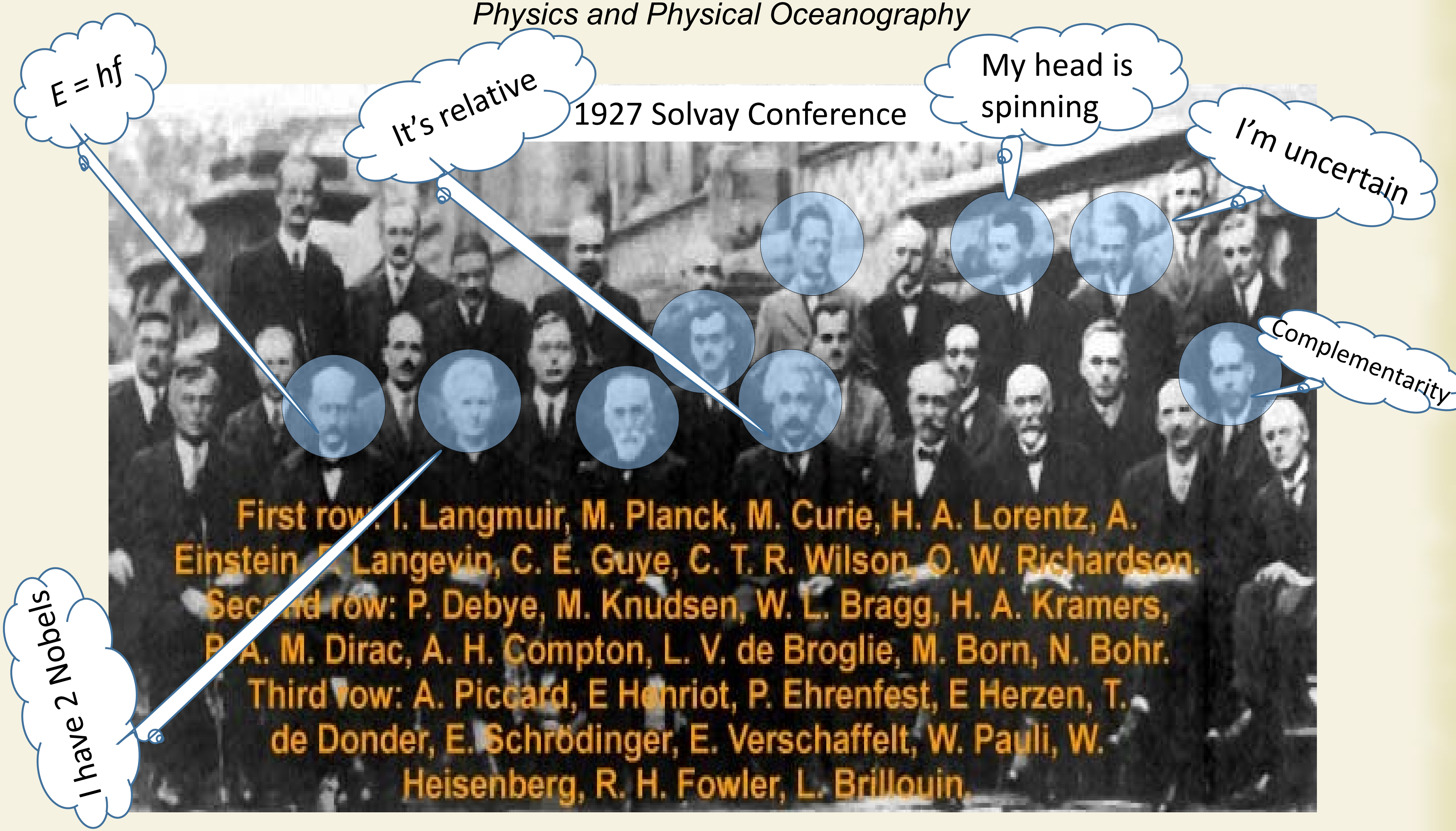
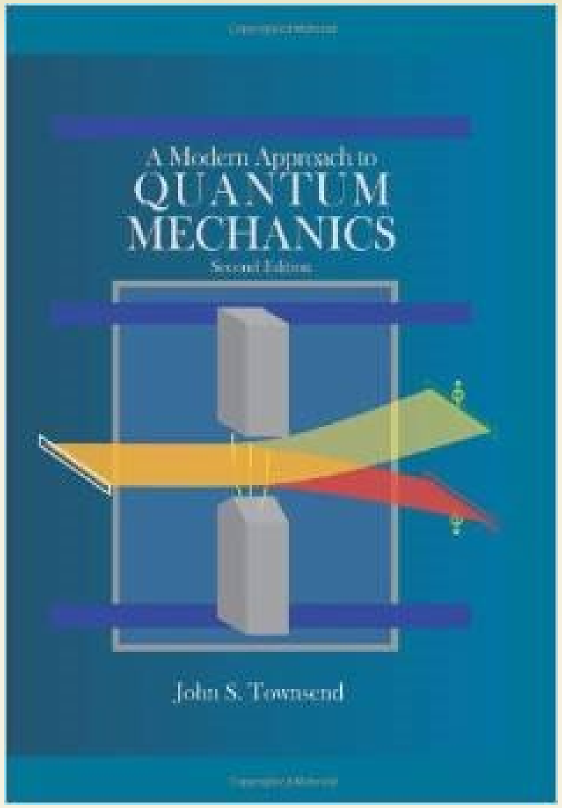
Introduction to Quantum Theory

Dr. Russell Herman
Physics and Physical Oceanography



Syllabus

- Website: http://people.uncw.edu/hermanr/qm/
- Grades
  Homework – 30%
  Papers – 10%
  3 Exams – 40%
  Final – 20%
- Office Hours: MTWRF, 10-11 AM
  Sartarelli Hall 2007J



Required Text:
Townsend, *J. A Modern Approach to Quantum Mechanics*, 2nd Ed., 2012.

Other Readings:
Susskind, L. *Quantum Mechanics, The Theoretical Minimum*, 2014.
Feynman, R. C.,
The Feynman Lectures on Physics, Vol. III, 1965 and
QED: The Strange Theory of Light and Matter, 1988.

See also - http://people.uncw.edu/hermanr/booklist.htm

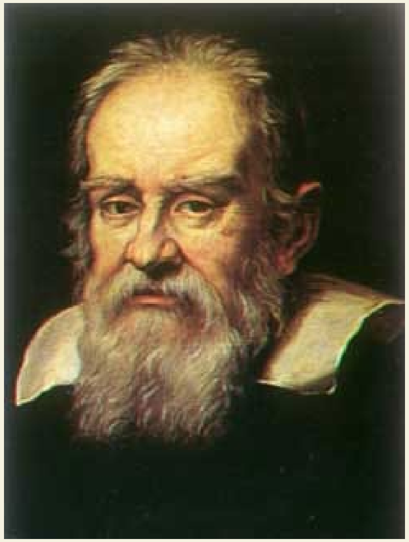
Time for Some Background

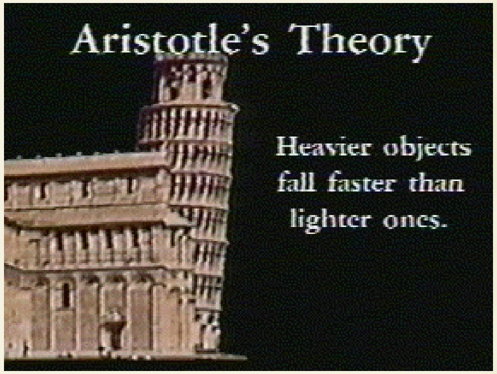
The Rise of Classical Physics

The Emergence of Physics - 1609

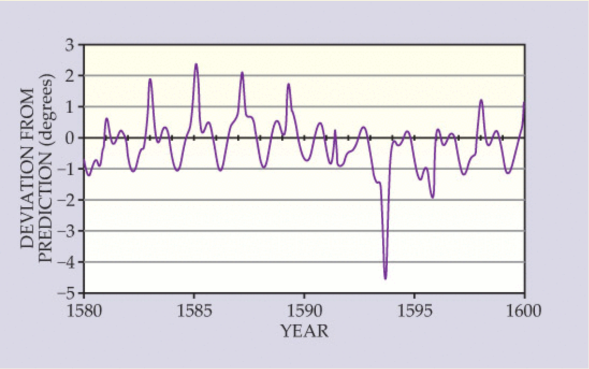
Nicolaus Copernicus (1773-1543)

Galileo Galilei (1564-1642)

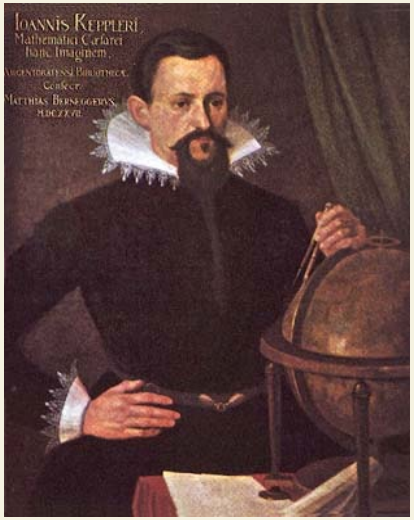


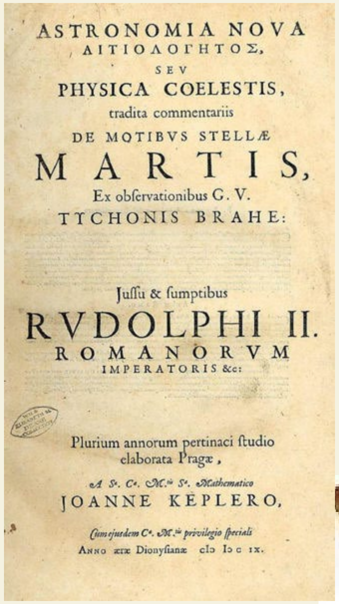


Tycho Brahe (1546-1601)



Johannes Kepler (1571-1630)





ASTRONOMIA NOVA
ΑΙΤΙΟΛΟΓΗΤΟΣ,
SEV
PHYSICA COELESTIS,
tradita commentariis
DE MOTIBVS STELLÆ
MARTIS,
Ex obſervationibus G. V.
TYCHONIS BRAHE:

Juſſu & ſumptibus
RVDOLPHI II.
ROMANORVM
IMPERATORIS &c:

Plurium annorum pertinaci ſtudio elaborata Pragæ,
A S. C. M.tis S. Mathematico
JOANNE KEPLERO,
Cum ejuſdem C. M.tis privilegio ſpeciali
Anno æræ Dionyſianæ clɔ Iɔ c ix.

<u>The great Martian catastrophe and how Kepler fixed it</u>

The Clockwork Universe

Sir Isaac Newton (1642-1727)

Principia (1687)

Philosophiae Naturalis Principia Mathematica (Mathematical Principles of Natural Philosophy)

Laws of Motion $d\mathbf{p}/dt = \mathbf{F}$

Law of Gravitation

Kepler's Laws Explained

Calculus (fluxions)

... Space and time are absolute ...

Determinism - Given **F**, predict **x** and **v**

Unification

... the force responsible for bodies falling on the Earth is the same as that causing the moon to follow its orbit.

Reformulations of $F = ma$

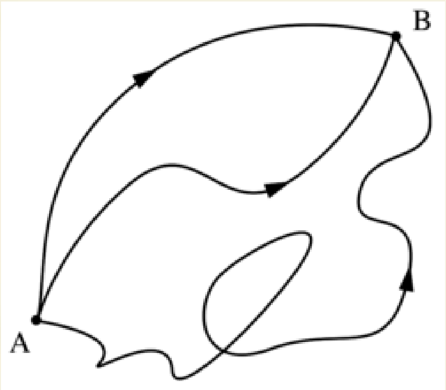
From Classical Dynamics

Euler (1707-1783) Variational Calculus

D'Alembert (1717-1783) Virtual Work

Lagrange (1736-1813) Lagrangian Mechanics

Hamilton (1805-1865) Hamiltonian Mechanics



Principles

Fermat's:	least time
d'Alembert's:	virtual work
Hamilton's:	least action

Define the action

$$S = \int_{t_1}^{t_2} L\,dt, \text{ for } L = T - V.$$

Require: $\delta S = 0.$

Then, $[F = ma]$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right) - \frac{\partial L}{\partial x} = 0.$$

Optics Vis viva Action Path Integrals
Fermat – Leibniz vs Maupertuis – Euler – Lagrange – Hamilton – ..... - Feynman
1662 1686 1744 1744 1788 1834 1948

Hamilton's Formulation

Phase Space ($q = x$, $p = m\, dx/dt$),

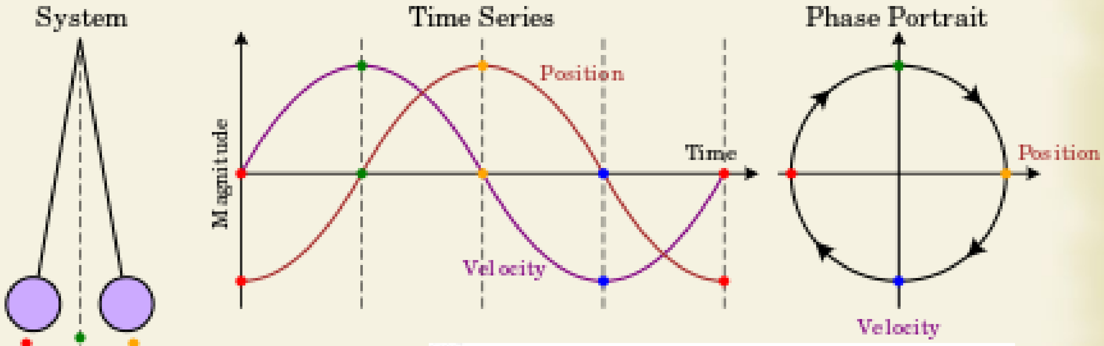
Initial (q, p) + 2nd Law => Motion for all t

Ex: Free particle, $p = \text{const}$ $\quad E = \dfrac{p^2}{2m} + \dfrac{1}{2}kq^2$

Harmonic Oscillator,

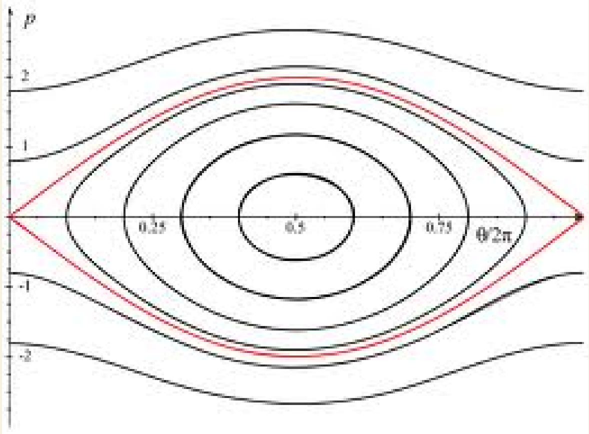
Energy Conservation $E = const$

Hamiltonian $H(p,q) = T(p) + V(q)$



Hamilton's Eqns <=> Newton's Laws

$$\boxed{\begin{aligned} \frac{dq}{dt} &= \frac{\partial H(q,p)}{\partial p} \\ \frac{dp}{dt} &= -\frac{\partial H(q,p)}{\partial q} \end{aligned}}$$



Poisson: For F(p,q), $\dfrac{d}{dt}F(p,q) = \dfrac{\partial F}{\partial q}\dfrac{\partial H}{\partial p} - \dfrac{\partial F}{\partial p}\dfrac{\partial H}{\partial q} = \{F, H\}$

Electricity and Magnetism

Magnetism (Lode stones, Compasses)

Electricity (static, lightning)

William Gilbert (1544-1603)
(amber, elecktron, magnetic + electric forces different)

Thomas Browne (1605-1682) ("electricity")

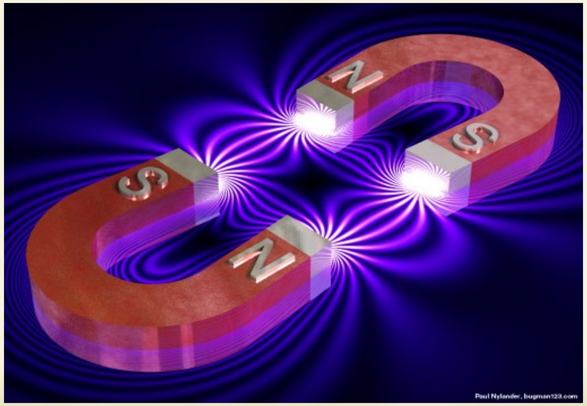
Benjamin Franklin (1706-1790) ("+/- electricity")

Henry Cavendish (1731-1810)

Charles-Augustin de Coulomb (1736-1806)

Luigi Galvani (1737-1798) (animal electricity)

Alessandro Volta (1745-1827) (electrochemical cell)



Electromagnetism

Hans Oersted (1777-1851)

1820 current deflects compass needles, made Al

André-Marie Ampère, (1775 - 1836)

“electrodynamics,” current carrying wires attract, telegraph

Georg Simon Ohm (1789-1854)

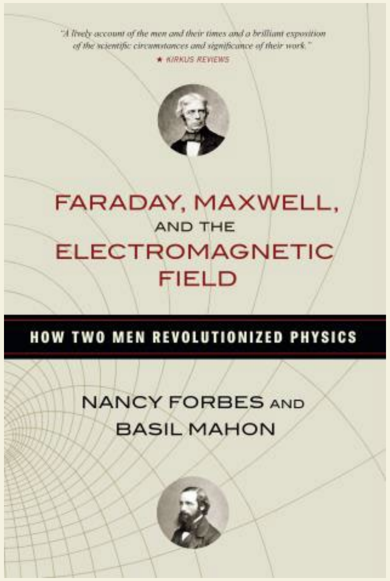
Ohm’s Law - 1827

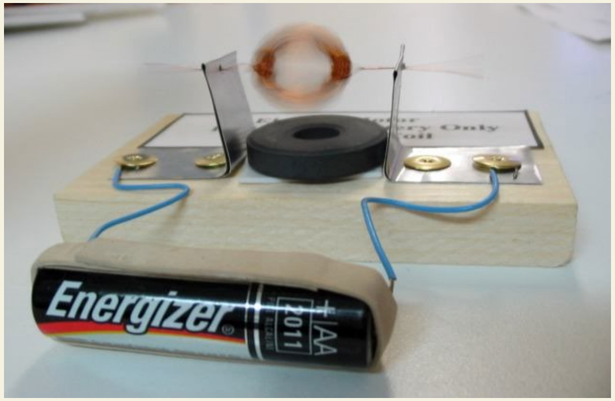
Joseph Henry (1797-1878)

electromagnetic induction, first motor, relays

Michael Faraday (1791-1867)

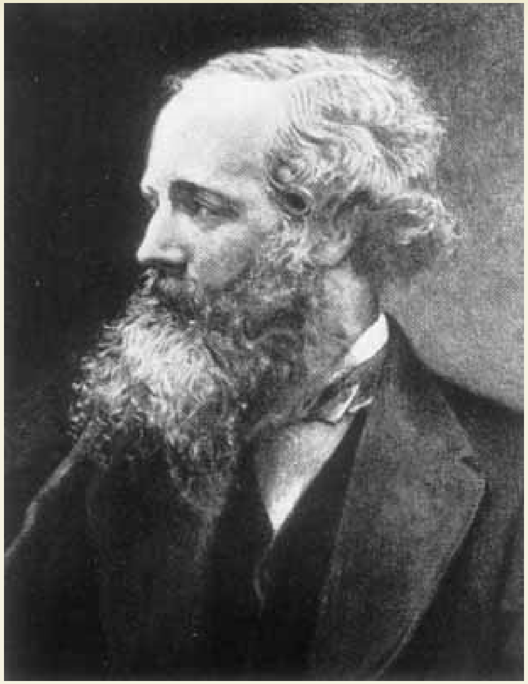
electrolysis, motors, induction coils, ...
Introduced concept of a field.





http://hilaroad.com/camp/projects/magnet.html

Electromagnetic Waves



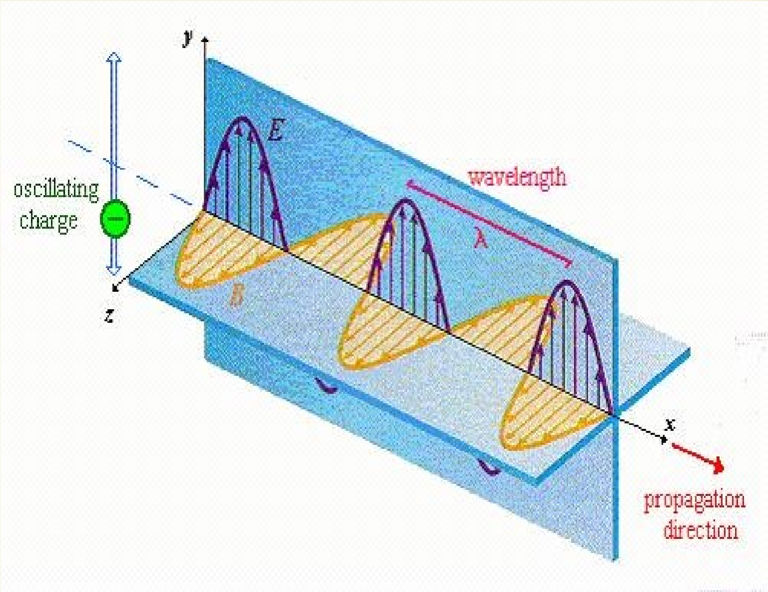
James Clerk Maxwell (1831-1879)

- Theory of electromagnetism – 1865.

  1873, Maxwell also used the quaternions of Hamilton (1843),
  1880s, Heaviside reduced the 20 PDEs –12 to 4,
  using symbolic vector calculus
  [independent of Josiah Gibbs, *Vector Analysis*, (1881-1884)]
  1890, Hertz presented other forms

- Predicted the electromagnetic waves - 1862.

  Electromagnetic waves travel: c = 299,792,458 m/s = 186,000 mi/s



The Maxwellians

George Francis FitzGerald (1851–1901),
Oliver Lodge (1851–1940) and Oliver Heaviside (1850)–1925)

Heinrich Hertz (1857-1894)

- Sent the first radio waves – 1888.
- Marconi (1874-1937), practical radio waves – 1897

What is the medium? - *Luminiferous Aether*

(supported by Thomson, Stokes, ...)

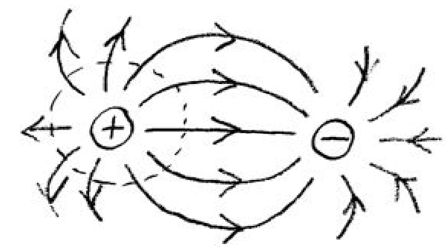
Maxwell's Equations

Not SI units!

JAMES CLERK MAXWELL'S EQUATIONS

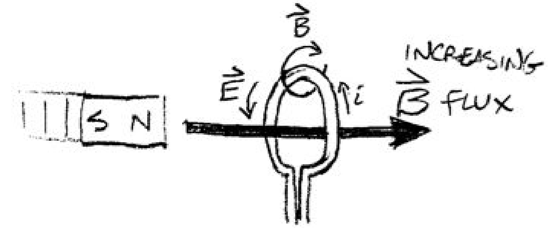
$$\boxed{\underbrace{\nabla\cdot\vec{E}}_{\text{DIVERGENCE OF }\vec{E}} = 4\pi\underbrace{\rho}_{\text{CHARGE DENSITY}}}$$

$\vec{E}$ DIVERGES OUT FROM POSITIVE CHARGES AND IN TOWARD NEGATIVE CHARGES. THE TOTAL FLUX OF $\vec{E}$ THROUGH ANY CLOSED SURFACE IS PROPORTIONAL TO THE CHARGE INSIDE.



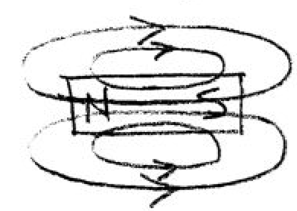
$$\boxed{\underbrace{\nabla\times\vec{E}}_{\text{CURL OF }\vec{E}} = -\underbrace{\frac{1}{c}}_{c:\ \text{SPEED OF LIGHT}}\underbrace{\frac{d\vec{B}}{dt}}_{\text{RATE }\vec{B}\text{ IS CHANGING}}}$$

$\vec{E}$ CURLS AROUND CHANGING $\vec{B}$ FIELDS (FARADAY'S LAW) IN A DIRECTION THAT WOULD MAKE A CURRENT THAT WOULD PRODUCE A $\vec{B}$ FIELD TO OPPOSE THE CHANGE IN $\vec{B}$ FLUX (LENZ'S LAW).



$$\boxed{\underbrace{\nabla\cdot\vec{B}}_{\text{DIVERGENCE OF }\vec{B}} = 0}$$

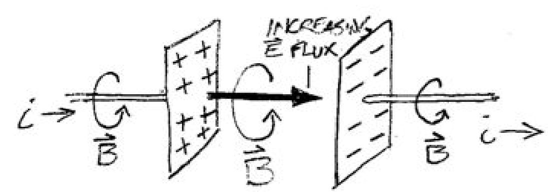
$\vec{B}$ NEVER DIVERGES. IT JUST LOOPS AROUND ON ITSELF.



$$\boxed{\underbrace{\nabla\times\vec{B}}_{\text{CURL OF }\vec{B}} = \frac{4\pi}{c}\underbrace{\vec{J}}_{\text{CURRENT DENSITY}} + \frac{1}{c}\underbrace{\frac{d\vec{E}}{dt}}_{\text{RATE }\vec{E}\text{ IS CHANGING}}}$$

c: SPEED OF LIGHT

$\vec{B}$ CURLS AROUND CURRENTS AND CHANGES IN $\vec{E}$ FIELDS



Gaussian Units

Unit Conversions

		Conversion	SI
Distance	cm	10^{-2}	m
Mass	g	10^{-3}	kg
Time	s	1	s
Force	dyne	10^{-5}	N
Energy	erg	10^{-7}	J
Power	erg/s	10^{-7}	W
Charge	esu	3.336×10^{-10}	C
Electric Potential	statvolt	299.79	V
Magnetic Field	Gauss	10^{-4}	T

- 1 eV = 1.6022×10^{-12} erg = 1.602×10^{-19} J
- 1 Ry = 13.6057 eV (ionization energy of hydrogen)
- 1 C = 2.9979×10^{9} esu, 1 statcoul = 1 esu
- 1 Å = 10^{-10} m
- 1 eV/c^2 = 1.7827×10^{-36} kg
- $(\mu_0\epsilon_0)^{-1/2} = 299,792,458$ m/s; $(\mu_0/\epsilon_0)^{1/2} \approx 377\Omega$

$$e^2 \text{ (Gaussian)} \rightarrow \frac{e^2}{4\pi\varepsilon_0} \text{ (SI)}$$

$$F_{\text{Coulomb}} = \frac{q_1 q_2}{r^2}$$

New Questions

- Waves – What is the medium?
  Michelson-Morley (1887) - could not detect it.
- Spectroscopy – Why the spectral lines?
- Blackbody Spectrum – Describe the dependence on λ.
- Lorentz Invariance – Explain speed of light in moving media.

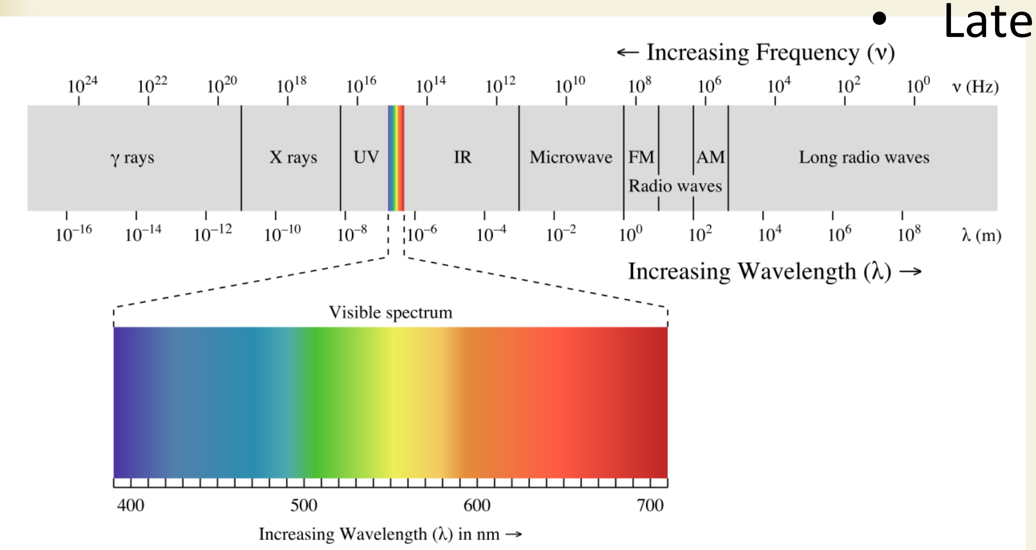
Led to Revolutions in Physics in the 1900s!

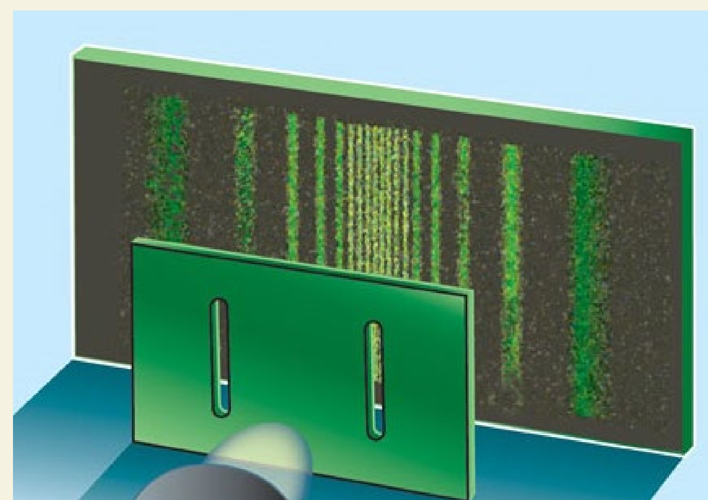
WAVES

What are waves?

- **Characteristics**
  - Wavelength, Frequency, Wavespeed
- **Behavior**
  - Superposition, Interference. Diffraction
  - Thomas Young, 1801, diffraction
  - 1817, the Académie des Sciences: diffraction would be the topic for the biannual physics *Grand Prix.* – proposed by corpuscular theorists.
  - Augustin-Jean Fresnel used Huygen's Principle, 1678.
  - Later – Airy, Stokes, Helmholtz, Kirchoff , and others.

$$\frac{1}{c^2}\frac{\partial^2 E}{\partial t^2} = \nabla^2 E$$





Spectroscopy

Robert Bunsen and Gustav Kirchhoff
developed the spectroscope, 1859.

Ionized gas gives off radiation

Johann Balmer 1885

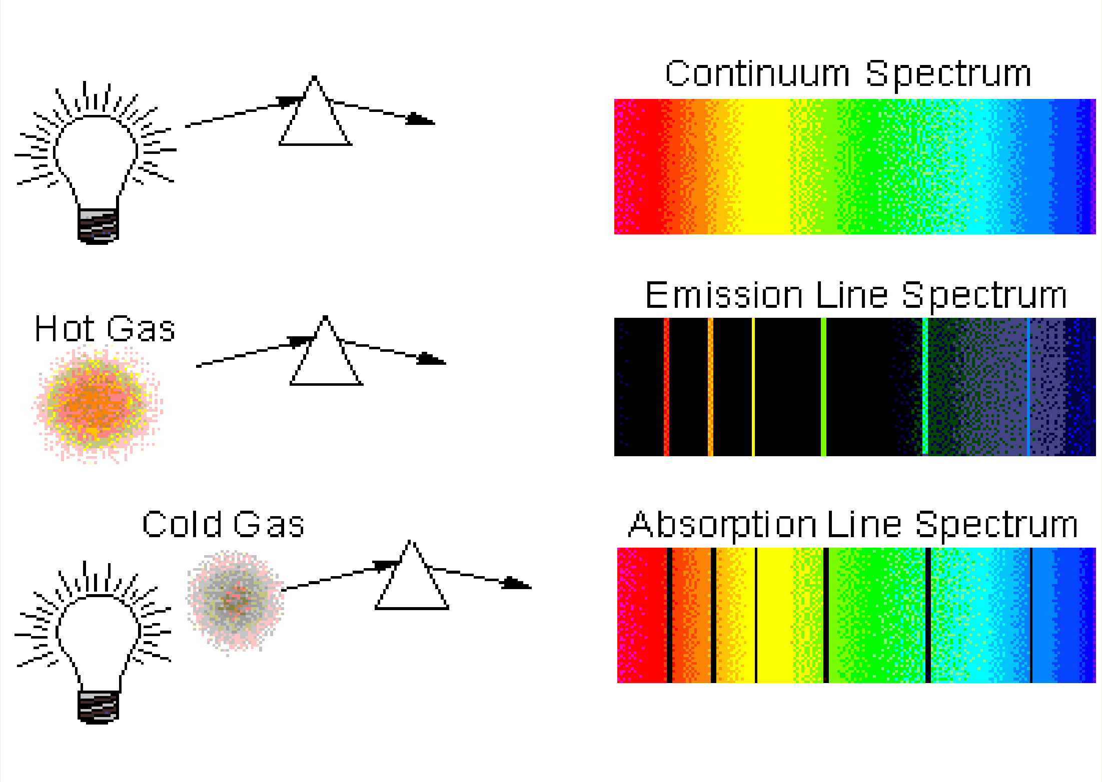
Spectral Lines: Hydrogen
410, 434, 486, 656 nm

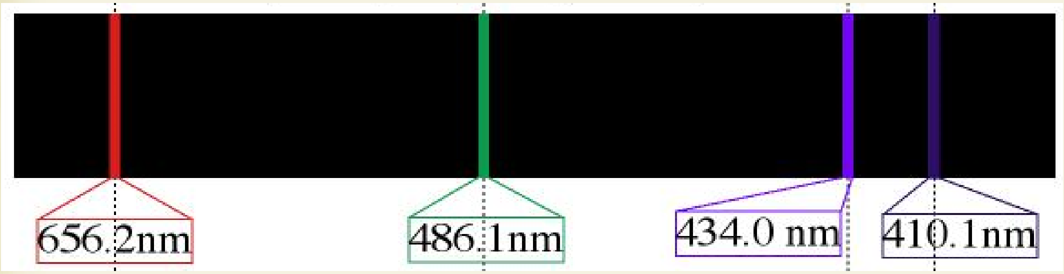
Derived Empirical Formula:

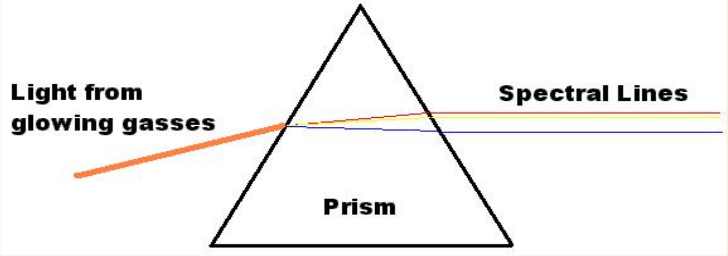
$$\lambda = \mathbf{R}\ (1/4 - 1/n^2)$$

Predicted 5th-7th lines

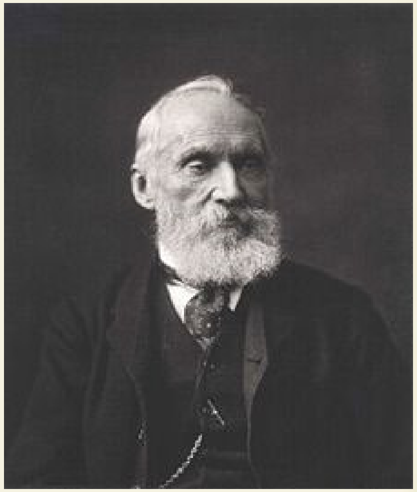
Lyman Series (1906-1914), ultraviolet
Paschen Series (1908), infrared







Laws of Thermodynamics



Engines: Watt, Carnot, Kelvin, Clausius

James Joule (1818-1889)
Mechanical Equivalent of Heat

Laws of Thermodynamics

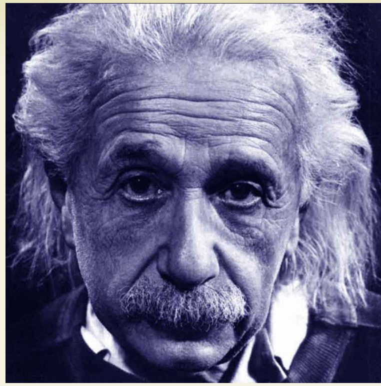
1. Adding heat energy or doing work on a body increases internal energy. (Energy conservation)
2. A body will not spontaneously get hotter. (Entropy and the Arrow of Time)

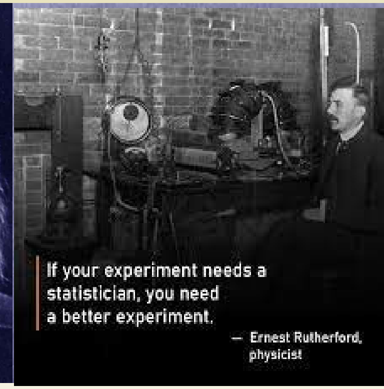
Joseph Stefan (1835-1893) and **Ludwig Boltzmann (1844-1906)**

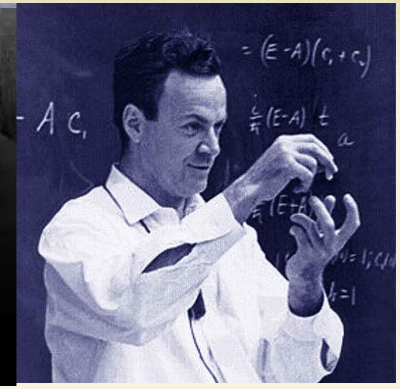
Heated bodies Radiate - Stefan-Boltzmann Law
Radiation from blackbody proportional to T^4.

$$\boxed{P = e\sigma A T^4}$$

Maxwell-Boltzmann Statistical Mechanics – *Bah Humbug!*



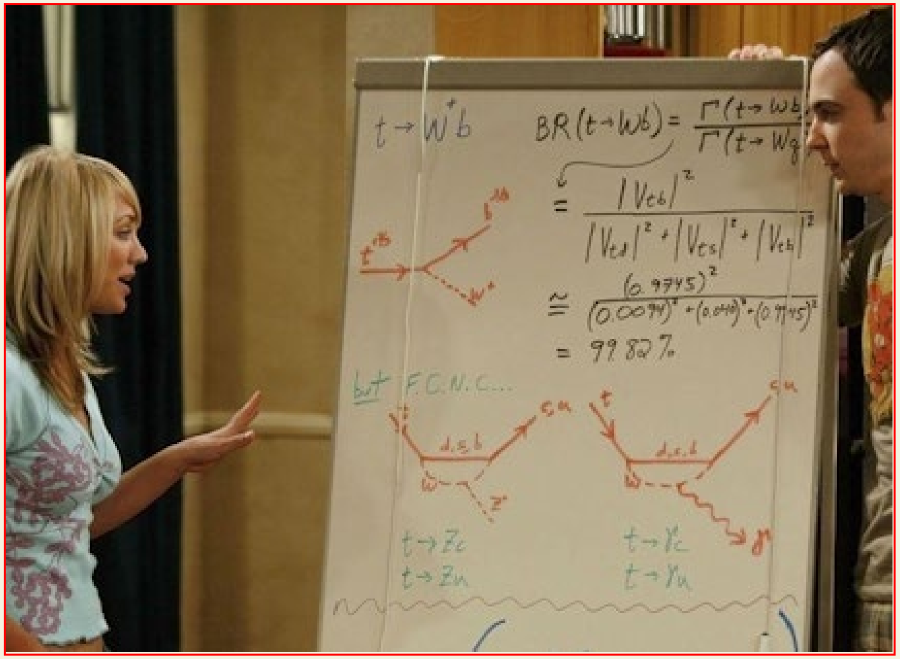




$$H\left|\psi\right\rangle = i\hbar\frac{\partial}{\partial t}\left|\psi\right\rangle$$

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4}T_{\mu\nu}$$

This is not your grandfather's physics



$$\left[i\hbar\gamma^{\mu}\partial_{\mu} - mc\right]\psi = 0$$

$$\Delta x \Delta p \geq \frac{\hbar}{2}$$

Physics Revolutions

Albert A. Michelson, (1852 – 1931)

"it seems probable that most of the grand underlying principles have been firmly established and that further advances are to be sought chiefly in the rigorous application of these principles ." - 1894

1897 **Joseph (J.J.) Thomson** measures electron, NP 1906
"plum-pudding" model of atom"

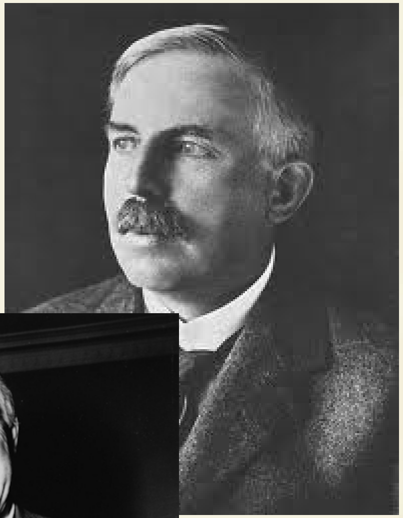
Radioactivity and the Atom

1895 - Wilhelm Röntgen discovers X-rays. NP 1901

1896 - Henri Becquerel discovers radioactivity NP 1903

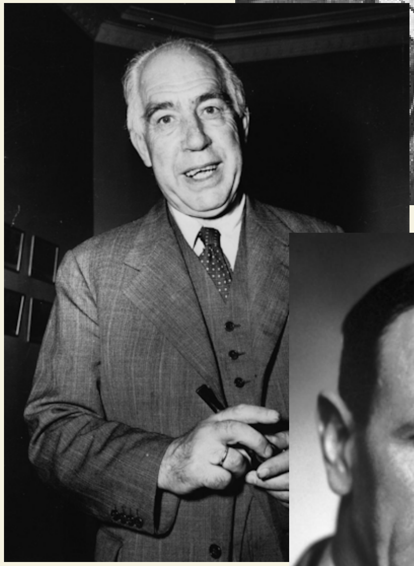
1897 - J.J. Thomson discovers the electron NP 1906.

1898 - Marie and Pierre Curie discover the first radioactive elements: radium and polonium NP 1903.



1899 - Ernest Rutherford divided radiation into alpha and beta rays NP 1908.

1900 - Pierre Curie observes gamma rays.



1911 - Ernest Rutherford discovers the atomic nucleus

1913 - Niels Bohr introduces the first atomic model, NP 1922 the mini solar system.

1913 - Hans Geiger invents counter for measuring radioactivity.

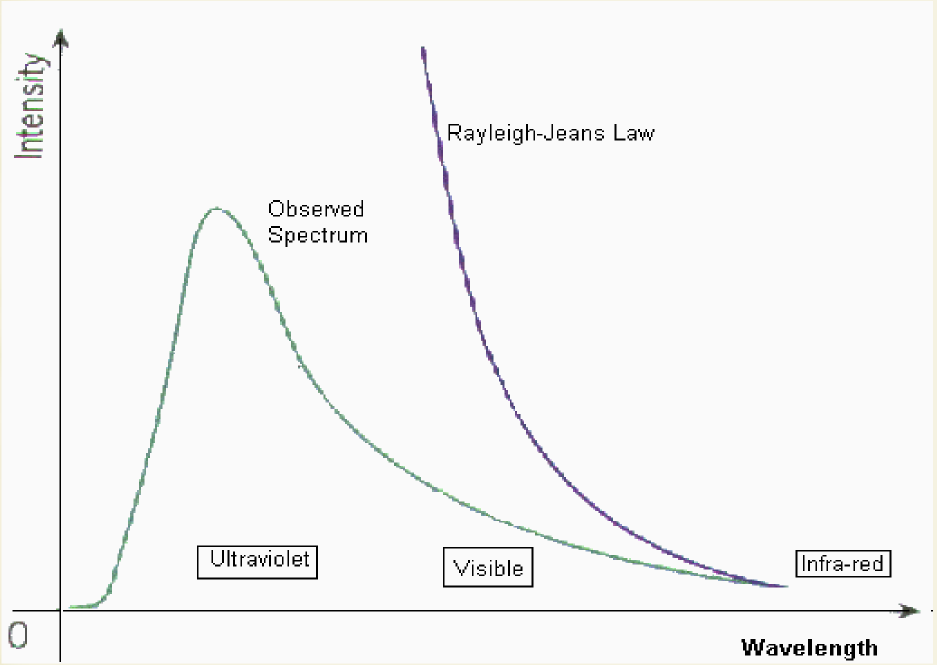
1920 - Ernest Rutherford discovered and named the proton.

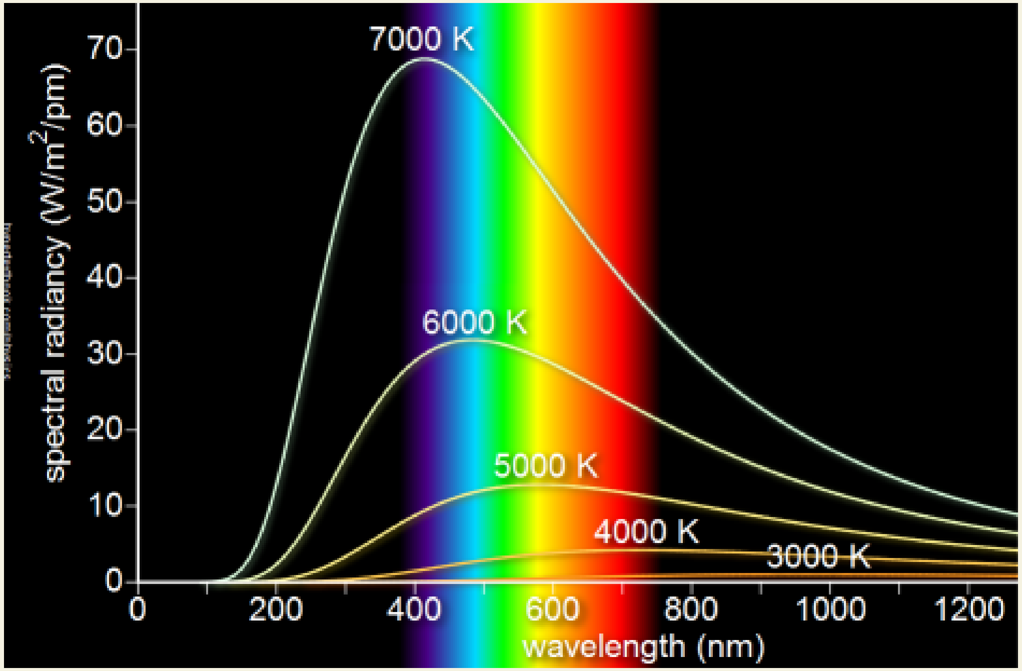
1932 - James Chadwick discovers the neutron. NP 1935

Blackbody Spectrum

Blackbody - a theoretical object that absorbs 100% of the radiation that hits it.

Wien's Law (1896) NP 1911 and **Rayleigh** NP 1904 **- Jeans Law** (1900/1905)





Ultraviolet (Rayleigh–Jeans) Catastrophe "... when you turn on your toaster, you are instantly fried by a massive gamma ray burst, since your little blackbody toaster should emit infinite energy at the shortest wavelengths."

For fixed T, monochromatic energy density becomes infinite at infinitely small wavelengths!

Quantum Theory

Max Planck NP 1918

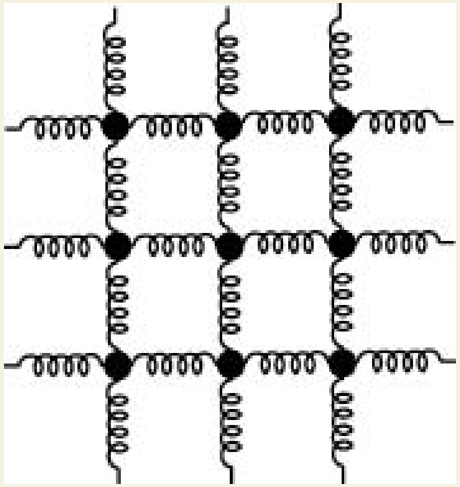
(Karl Ernst Ludwig Max Planck 1858-1947)

What makes hot solids glow different colors?

Nov. 1900 – he used Boltzmann's theory.

oscillators can only vibrate at discrete frequencies:

$$E_n = n(hf),\ n = 1,2,3\ \ldots.$$



Thus, the energy difference

$$\Delta E = hf,$$

where Planck's constant is given by

$$h = 6.63 \times 10^{-34}\ Js$$

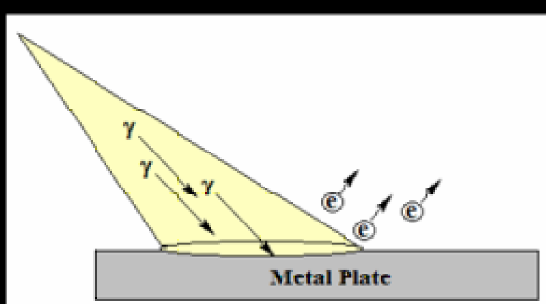
Albert Einstein - 1905

Received 18 March and published 9 June,

Photoelectric Effect

Light can cause currents

- Electrons can be ejected from irradiated metal plates.
- Light can be act like either particles (quanta) or waves.
- Extended Planck's ideas of energy quantization.
- Lead to explanation of electromagnetic spectra,
- Lead to the development of lasers, transistors and other applications.

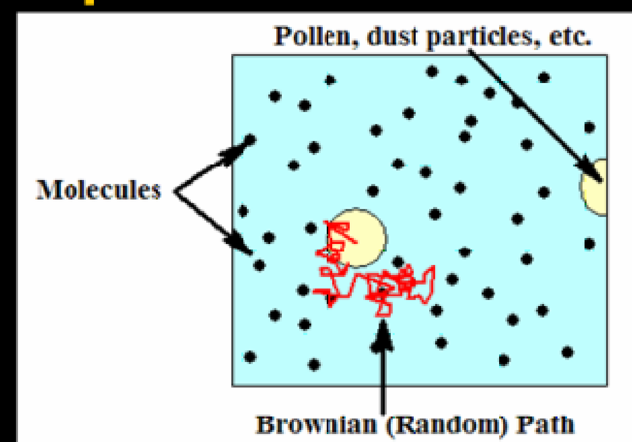


Received 11 May and published 18 July

Brownian Motion

the random movement of particles suspended in a fluid

- Explained the observations credited to Robert Brown, 1827
- Predicted molecular motion and size through the effects of collisions with larger particles
- Einstein's work lead to an acceptance of molecular theory



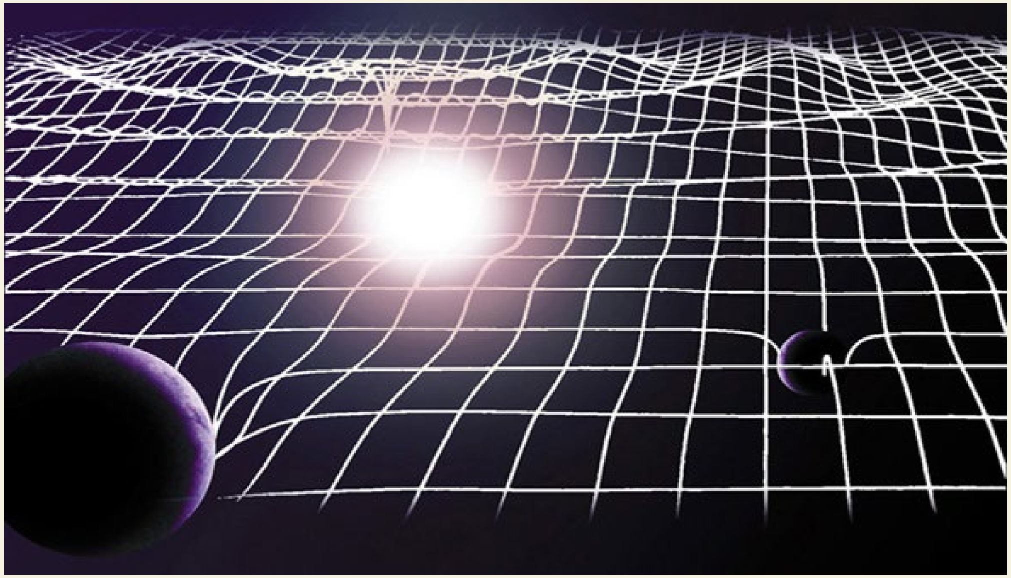
Special Relativity: received on 30 June and published 26 September

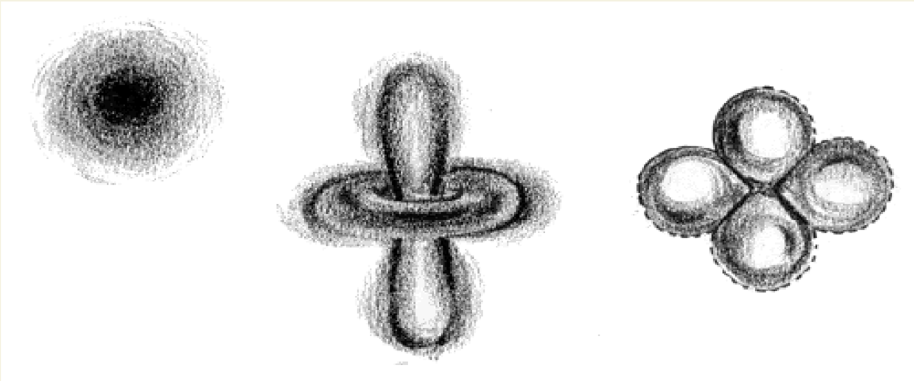
Mass-Energy Equivalence: received September 27 and published 21 November

Paradigm Shifts – *in progress*

Relativity

Space and Time are not absolute, there is no preferred frame.
and not Euclidean





$$\Delta t = \gamma \Delta \tau$$

$$\Delta x \Delta p \geq \frac{\hbar}{2}$$

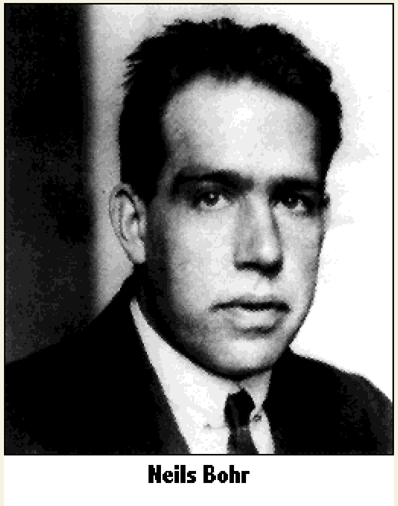
Quantum Mechanics

Loss of Determinism:

Determinism: If we knew all of the initial conditions, we couldn't predict the exact position and velocity of an electron.

http://www.zamandayolculuk.com/cetinbal/AE/Deformation.jpg

Bohr's Atom - 1913



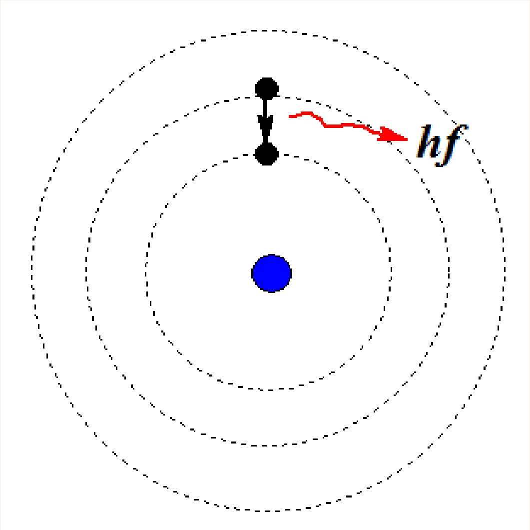
Niels Bohr (1885-1962)

Accelerating electrons radiate at specific energies.

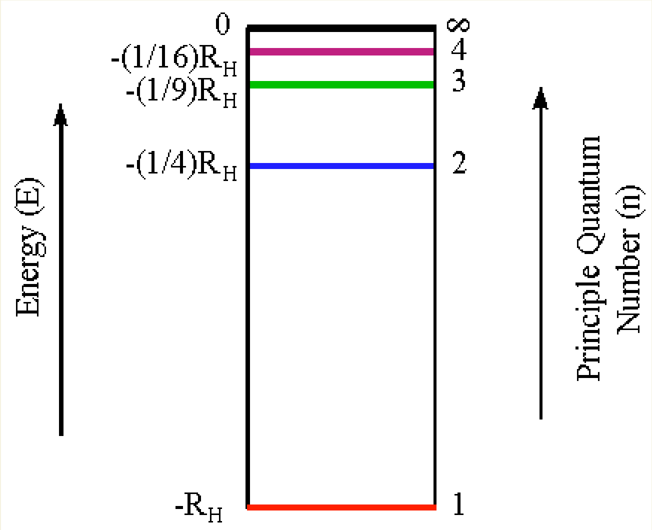
Are atoms stable?

Assume angular momentum is quantized.

Derived Balmer’s formula.



$$E_n = R_H\left(\frac{1}{m^2} - \frac{1}{n^2}\right)$$



Early History - Quantum Mechanics

1900 - Planck Explains Blackbody Radiation NP 1906

1905 - Einstein - the Photoelectric Effect, Photons NP 1921

1913 - The Bohr Model for Hydrogen NP 1922

1916 - Confirmation of photon, Millikan NP 1923

1922 – Stern-Gerlach Experiment NP 1943

1923 - Compton NP 1927 Effect – X-Ray Scattering

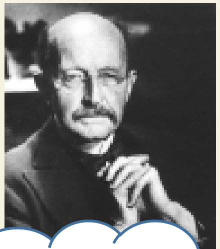
1924 – de Broglie NP 1929 - Particles Behave Like Waves

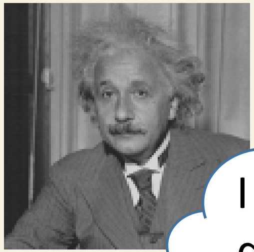
1925 - Matrix Mechanics – Heisenberg NP 1932, Born NP 1954, Jordan
- Pauli Principle NP 1935
- Uhlenbeck and Goudsmit, spinning particles

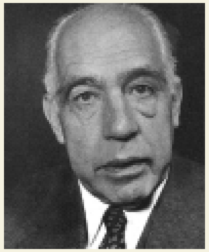
1926 - Wave Mechanics - Schrödinger NP 1933

1927 - The Uncertainty Principle – Heisenberg
- Davisson NP 1937 –Germer, Thomson NP 1937 - Verified de Broglie's idea

1928 - Relativistic Quantum Mechanics - Dirac NP 1933







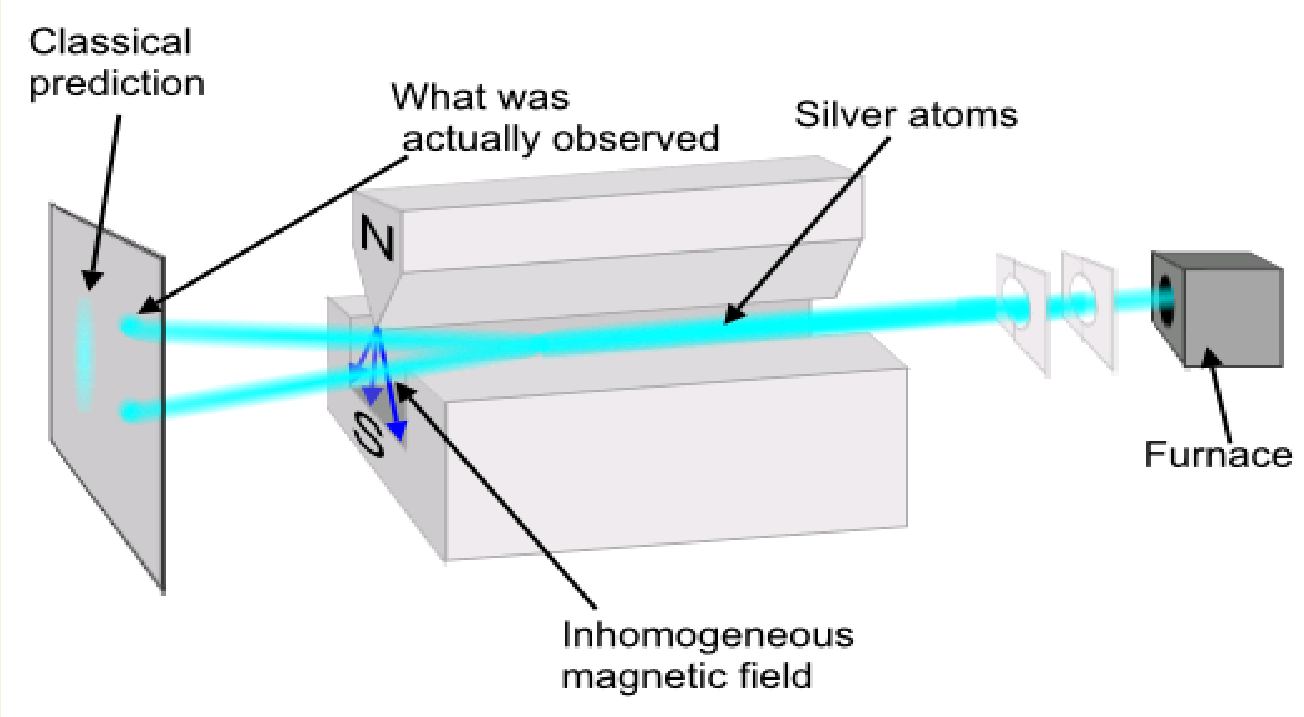
Stern-Gerlach Experiment - 1922

Otto Stern (1888-1969)

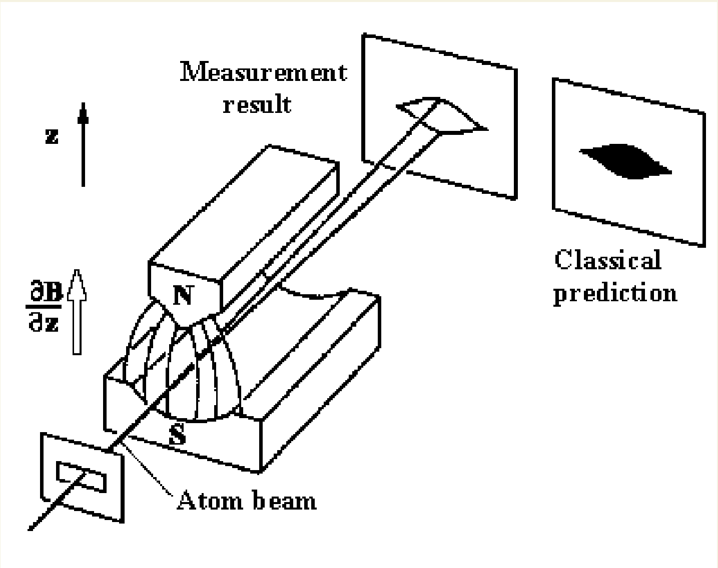
Walther Gerlach (1889-1979) – Was it the cigar?

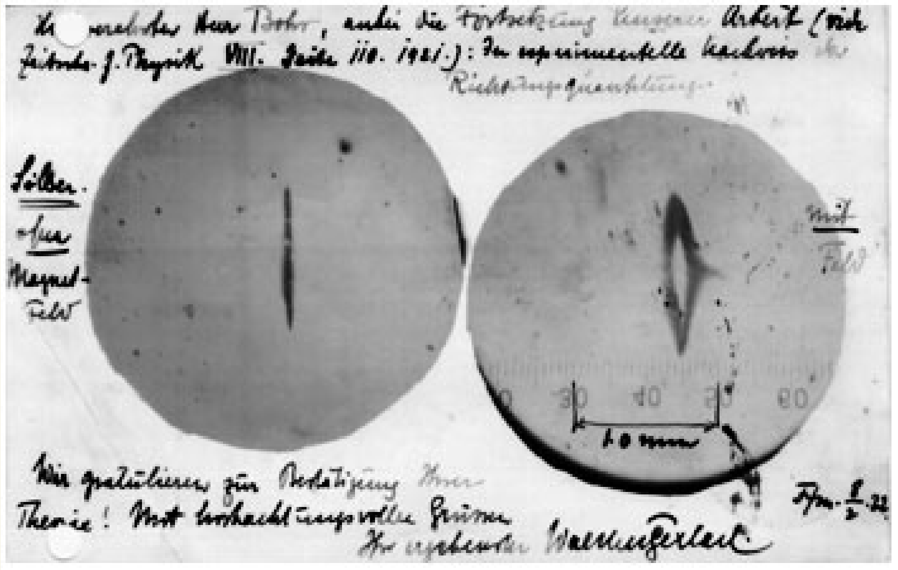
Ag atoms sent through inhomogenous magnetic field.

Demonstration of space quantization.



Stern-Gerlach Results





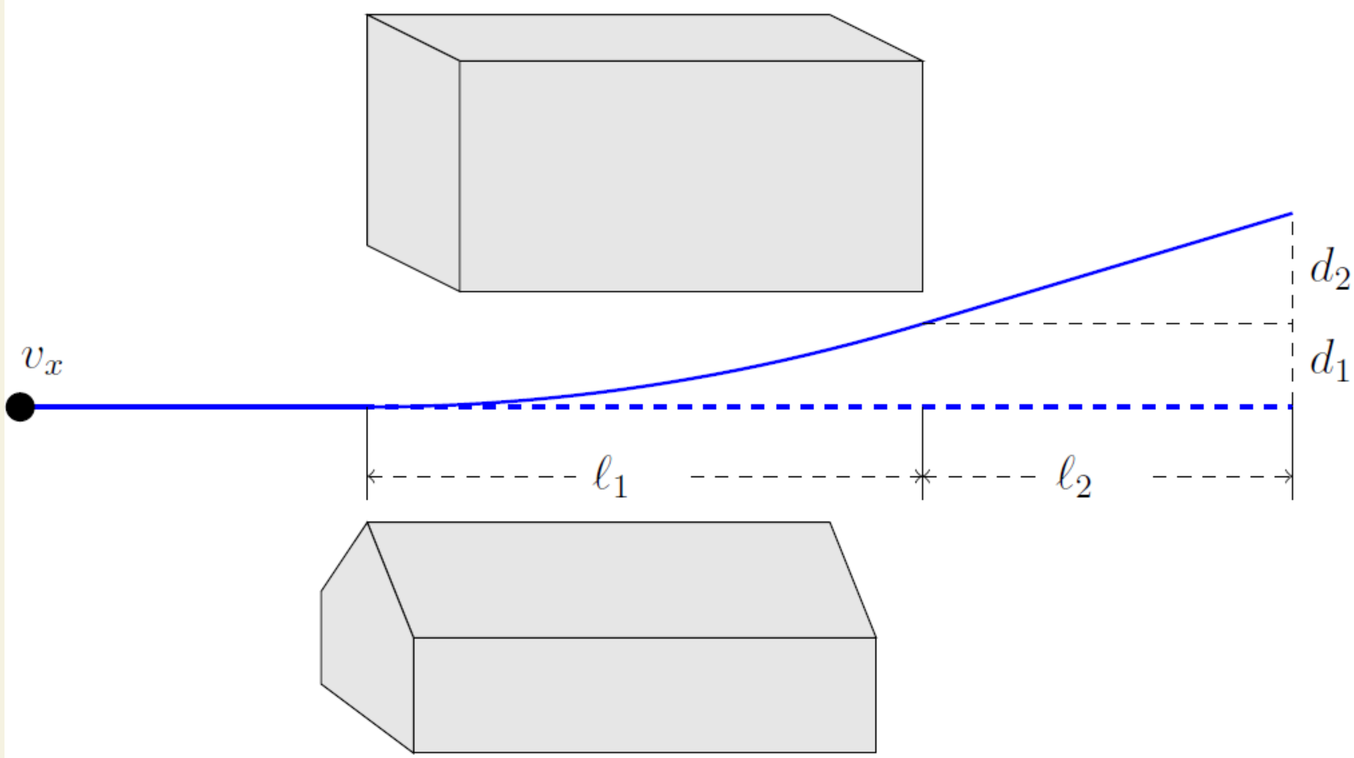
Test of classical vs quantum theory of angular momentum (**L**).

L = 0 or *L = 1* – No splitting for *L = 0*?

Uhlenbeck and Goldsmit (1925,1926) proposed intrinsic spin.
- to explain the anomalous Zeeman effect,
(the splitting of spectral lines in a magnetic field).

Stern-Gerlach Particle Path – Problem 1

$$a_z = \frac{\mu_B}{M}\frac{\partial B_z}{\partial z} \qquad \tfrac{1}{2}Mv_x^2 = 2k_BT$$



$$x = v_x t_1 = \ell_1 \qquad x = v_x t_2 = \ell_2$$

$$z = \tfrac{1}{2}a_z t_1^2 = d_1 \qquad z = v_z t_2 = d_2$$

Stern-Gerlach Thought Experiments

1

2

3