PHY 444 Exam II **Name ________________________**

Instructions:

- Place your name on all of the pages.
- Do all of your work in this booklet. Do not tear off any sheets.
- Show all of your steps in the problems for full credit.
- Be clear and neat in your work. Any illegible work, or scribbling in the margins, will not be graded.
- Put a box around your answers when appropriate.
- If you need more space, you may use the back of a page or use the **Extra Space** and write *On back of page #* in the problem. **No other scratch paper is allowed.**

Try to answer as many problems as possible. Provide as much information as possible. Show sufficient work or rationale for full credit. Remember that some problems may require less work than brute force methods.

If you are stuck, or running out of time, indicate as completely as possible, the physics and steps you would take to tackle the problem. Also, indicate any relevant information that you would use.

Pace yourself – do not spend more than 15 minutes per page on your first pass.
Pay attention to the point distribution. Not all problems have the same weight.

Page	Pts	Score
1	16	
2	12	
3	18	
4	9	
Total	55	

Spin-1 Matrices: $\hat{S}_x = \frac{\hbar}{\sqrt{2}}\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \quad \hat{S}_y = \frac{\hbar}{\sqrt{2}}\begin{pmatrix} 0 & -i & 0 \\ i & 0 & -i \\ 0 & i & 0 \end{pmatrix}, \quad \hat{S}_z = \hbar\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}.$

Eigenvectors of S_x : $\frac{1}{2}\begin{pmatrix} 1 \\ \pm\sqrt{2} \\ 1 \end{pmatrix}, \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$ Eigenvectors of S_y : $\frac{1}{2}\begin{pmatrix} \mp 1 \\ \sqrt{2}i \\ \pm 1 \end{pmatrix}, \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$

$\hbar = 1.05457 \times 10^{-34}\ \text{Js} = 6.582 \times 10^{-16}\ \text{eVs}$

Bonus 1: A proton can hypothetically violate energy conservation by emitting or absorbing a virtual π meson, rest mass = 135 MeV/c^2, as long as this process occurs in a time Δt consistent with the uncertainty principle. Estimate this time window using the rest mass of the π meson.

Bonus 2: Name one of the winners of the 2022 Nobel Prize. ______________________

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1. (10 pts) Answer the following:
   a. Write the Schrödinger equation for the evolution of the state $\left|\psi(t)\right\rangle$?

   b. What conditions does one need for $\left\langle A\right\rangle$ to be a constant of motion?

   c. Let $\left|\psi(t)\right\rangle = \hat{U}(t)\left|\psi(0)\right\rangle$. What is $\hat{U}(t)$ in terms of a time-independent Hamiltonian?

   d. The spin of a spin-1/2 particle is aligned in the *y*-direction in an external magnetic field oriented in the positive *x*-direction. Describe its motion with respect to the *x*-axis.

   e. Complete the uncertainty relation: $\Delta S_x \Delta S_y \geq$ _______

2. (6 pts) Compute and simplify the following:
   a. Simplify $[\hat{J}_z, \hat{J}_x \hat{J}_y] =$

   b. $S_+\left|\frac{5}{2}, -\frac{1}{2}\right\rangle = ?$

3. (5 pts) Consider the state $|\psi\rangle = |3,-1\rangle$. Evaluate and simplify:
   a. $\hat{J}^2|\psi\rangle =$
   b. $\hat{J}_z|\psi\rangle =$
   c. $\hat{J}_-\hat{J}_+|\psi\rangle$.

4. (4 pts) Compute the following for single particle states.
   a. $\left\langle \frac{3}{2},-\frac{1}{2}\right| S_- \left|\frac{3}{2},\frac{1}{2}\right\rangle$
   b. ${}_y\left\langle \frac{1}{2},-\frac{1}{2}\middle|\frac{1}{2},-\frac{1}{2}\right\rangle_z$

5. (4 pts) Bob and Alice created a spin-0 state from their spin-1/2 particles and Alice takes hers home. That evening, Eve sneaks into Bob's lab and applies the Hadamard transformation, $\hat{\mathbf{H}} = \frac{1}{\sqrt{2}}(\boldsymbol{\sigma}_x + \boldsymbol{\sigma}_z)$, to Bob's particle without him knowing.
   a. When Eve leaves, what state is Bob's and Alice's particle in?
   b. The next morning Alice measures a $|+z\rangle$ state. What is the probability that Bob then measures a $|-z\rangle$ state?

6. (13 pts) A spin-1 particle is in the normalized state $|\psi\rangle \rightarrow \frac{1}{\sqrt{6}}\begin{pmatrix} i \\ 2 \\ 1 \end{pmatrix}$ in the $\hat{S}_z$ basis.

   a. What is the probability that a measurement of $\hat{S}_z$ will give 0?

   b. What is $\langle S_z \rangle$?

   c. Find ΔS_x.

   d. What is the probability that a measurement of $\hat{S}_x$ will give $-\hbar$?

   e. What is $\langle S_x \rangle$?

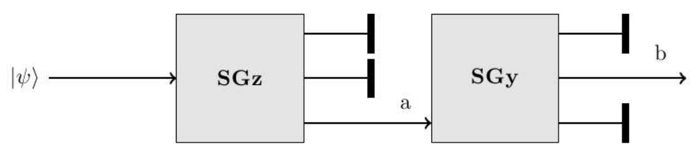
7. (5 pts) Consider the Hamiltonian $\hat{H} = 3\hat{\sigma}_z + 4\hat{\sigma}_x \rightarrow \begin{pmatrix} 3 & 4 \\ 4 & -3 \end{pmatrix}$ acting on spin-½ states.

   a. What are the energy eigenstates ($|E_1\rangle, |E_2\rangle$) and eigenvalues (E_1, E_2)?

   b. If $|\psi(0)\rangle = \frac{1}{\sqrt{2}}\left(|E_1\rangle + |E_2\rangle\right)$, what is $|\psi(t)\rangle$ in simplest form?

8. (3 pts) Consider a basis of states consisting of three spin-½ particles. Find an expression for the state $\left|\frac{3}{2},\frac{1}{2}\right\rangle$ in terms of three particle states, such as $\left|-z,+z,+z\right\rangle$, by applying the total spin lowering operator $\hat{S}_- = \hat{S}_{1-} + \hat{S}_{2-} + \hat{S}_{3-}$ to both sides of the expression $\left|\frac{3}{2},\frac{3}{2}\right\rangle = \left|+z,+z,+z\right\rangle$.

9. (6 pts) A beam of spin-1 particles in state $\left|\psi\right\rangle = N\left(\left|1,1\right\rangle + \left|1,0\right\rangle + 2\left|1,-1\right\rangle\right)$ enters the Stern-Gerlach devices as depicted below with two of the three channels blocked after each transition.



a. Determine the normalization constant, *N*.

b. Determine the probability that particles follow channel a: ______

c. Determine the probability that particles pass from a to b: ______ [First writing the appropriate probability amplitude indicating the basis needed. (For example: ${}_z\left\langle 1,1 | 1,1\right\rangle_x$.]

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Extra Space