PHY 444 Exam II **Name** ________________________

Instructions:

- Place your name on all of the pages.
- Do all of your work in this booklet. Do not tear off any sheets.
- Show all of your steps in the problems for full credit.
- Be clear and neat in your work. Any illegible work, or scribbling in the margins, will not be graded.
- Put a box around your answers when appropriate.
- If you need more space, you may use the back of a page and write *On back of page #* in the problem. **No other scratch paper is allowed.**

Try to answer as many problems as possible. Provide as much information as possible. Show sufficient work or rationale for full credit. Remember that some problems may require less work than brute force methods.

If you are stuck, or running out of time, indicate as completely as possible, the physics and steps you would take to tackle the problem. Also, indicate any relevant information that you would use.

Pace yourself – do not spend more than 15 minutes per page on your first pass.

Pay attention to the point distribution. Not all problems have the same weight.

Page	Pts	Score
1	15	
2	15	
3	10	
4	10	
Total	50	

Spin-1 Matrices: $\hat{S}_x = \frac{\hbar}{\sqrt{2}}\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \quad \hat{S}_y = \frac{\hbar}{\sqrt{2}}\begin{pmatrix} 0 & -i & 0 \\ i & 0 & -i \\ 0 & i & 0 \end{pmatrix}, \quad \hat{S}_z = \hbar\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}.$

Eigenvectors of S_x: $\frac{1}{2}\begin{pmatrix} 1 \\ \pm\sqrt{2} \\ 1 \end{pmatrix}, \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$ Eigenvectors of S_y: $\frac{1}{2}\begin{pmatrix} \mp 1 \\ \sqrt{2}i \\ \pm 1 \end{pmatrix}, \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$

Bonus: If $\hat{f}, \hat{g}$ are Hermitian, then what can you say about α to guarantee that $\alpha[\hat{f}, \hat{g}]$ is also Hermitian?

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1. (10 pts) Compute the following:
    a. $[\hat{J}_z, \hat{J}_y \hat{J}_x] = ?$

    b. $S_- \left| \frac{5}{2}, \frac{3}{2} \right\rangle = ?$

    c. Write $\hat{S}_y$ in terms of $\hat{S}_+$ and $\hat{S}_-$.

    d. What is the Pauli spin matrix σ_y ?

    e. What conditions does one need for $\langle A \rangle$ to be a constant of motion?

2. (5 pts) Consider the operator $\hat{J}_+ \hat{J}_-$.
    a. Derive an expression for $\hat{J}_+ \hat{J}_-$ in terms of $\hat{J}^2$ and $\hat{J}_z$.

    b. Use the expression in part a. to simplify $\hat{J}_+ \hat{J}_- \left| \frac{5}{2}, -\frac{1}{2} \right\rangle$.

3. (5 pts) The matrix representation of $\hat{S}_x$ in the $\hat{S}_z$ basis for spin-½ particles is given by $S_x = \frac{\hbar}{2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$. Find the eigenvalues and eigenvectors of S_x.

4. (10 pts) A spin-1 particle is in the state $|\psi\rangle \to \frac{1}{3}\begin{pmatrix} 2 \\ -1 \\ 2i \end{pmatrix}$ in the $\hat{S}_z$ basis.

   a. What is the probability that a measurement of $\hat{S}_z$ will give $+\hbar$?

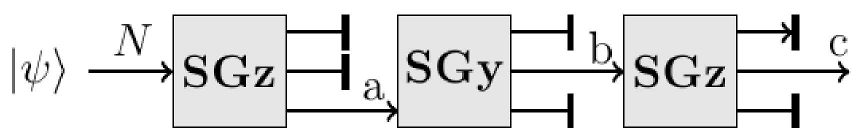
   b. What is $\langle S_z \rangle$?

   c. What is the probability that a measurement of $\hat{S}_x$ will give $+\hbar$?

   d. What is $\langle S_x \rangle$?

5. (4 pts) Consider three spin-½ particles in the state $\left|\frac{3}{2},-\frac{3}{2}\right\rangle = \left|-z,-z,-z\right\rangle$. Apply the total spin raising operator $\hat{S}_+ = \hat{S}_{1+} + \hat{S}_{2+} + \hat{S}_{3+}$ to this state to obtain an expression for the **normalized** state $\left|\frac{3}{2},-\frac{1}{2}\right\rangle$ as a sum of three particle states such as $\left|-z,+z,-z\right\rangle$.

6. (6 pts) A beam of spin-1 particles enters the sequence of Stern-Gerlach devices as depicted below with two of the three channels blocked after each transition. Find the requested probabilities indicating the bases needed. For example: ${}_z\langle 1,1|1,1\rangle_x$.



   a. Probability that particles pass from a to b: ______

   b. Probability that particles pass from b to c: ______

   c. How many of $N = 800$ particles do you expect to reach c? ______

7. (5 pts) Consider the Hamiltonian $\hat{H} = A\hat{S}_x$ acting on spin-½ states for real constant A.
   a. What are the energy eigenstates ($|E_1\rangle, |E_2\rangle$) and eigenvalues (E_1, E_2).

   b. If $|\psi(0)\rangle = \frac{1}{\sqrt{2}}\left(|E_1\rangle + |E_2\rangle\right)$, what is $|\psi(t)\rangle$ in simplest form?

8. (5 pts) An electron in state $|\psi(0)\rangle = |+\mathbf{x}\rangle$ is placed in a uniform field $\mathbf{B} = B_0\mathbf{j}$.
   a. Write the Hamiltonian in terms of m, e, and B_0 and a spin operator.

   b. Describe the expectation values $\langle S_x \rangle$, $\langle S_y \rangle$, and $\langle S_z \rangle$ as functions of time and the motion of the electron in the magnetic field.

Extra Space