PHY 444 Exam I **Name ________________________**

Instructions:

- Place your name on all of the pages.
- Do all of your work in this booklet. Do not tear off any sheets.
- Show all of your steps in the problems for full credit.
- Be clear and neat in your work. Any illegible work, or scribbling in the margins, will not be graded.
- Put a box around your answers when appropriate.
- If you need more space, you may use the back of a page and write *On back of page #* in the problem or use the **Extra Page** that's attached. **No other scratch paper is allowed.**

Try to answer as many problems as possible. Provide as much information as possible. Show sufficient work or rationale for full credit. Remember that some problems may require less work than brute force methods.

If you are stuck, or running out of time, indicate as completely as possible, the physics and steps you would take to tackle the problem. Also, indicate any relevant information that you would use.

Pace yourself – do not spend more than 15 minutes per page on your first pass.

Pay attention to the point distribution. Not all problems have the same weight.

Page	Pts	Score
1	16	
2	13	
3	12	
4	9	
Bonus		
Total	50	

Bonus:

a. Let $|\psi\rangle \xrightarrow[S_z]{} \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -i \end{pmatrix}$ be the representation of a wavefunction in the S_z basis.
What is the matrix representation of the wavefunction in the S_x basis?

b. When can a unitary operator $\hat{U}$ also be Hermitian?

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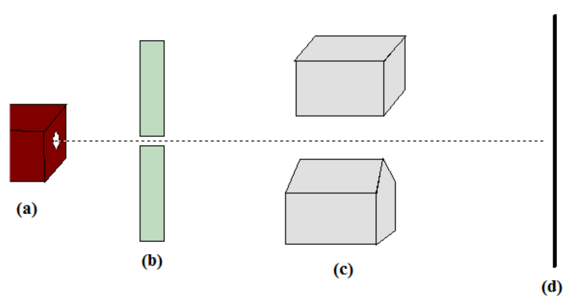
1. (2 pts) Consider the matrix $A = \frac{1}{2}\begin{pmatrix} 1 & i \\ -i & -1 \end{pmatrix}$. Is it
   a. Hermitian? __________
   b. Unitary? __________

2. (2 pts) Why was the Stern-Gerlach experiment of 1922 important?

3. (3 pts) In the below Stern-Gerlach device the lower magnet is marked S and the upper is N. A beam of silver atoms is sent through the device. Show the path of the beam starting from (a) and indicate what is seen on the screen with appropriate labels and spins.



4. (5 pt) Use the force on the magnetic moment of a silver atom to determine the field gradient (in T/m) needed to deflect a beam of silver atoms 0.16 mm from the dotted line as they travel 3.50 cm through the magnetic field at an average speed of 750.0 m/s. [$\mu_B = \frac{e\hbar}{2m_e c} = 9.274\times10^{-24}$ J/T, $M_{Ag} = 1.791\times10^{-25}$ kg]

5. (4 pts) Evaluate $e^{iA\theta}$ for $A = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ in the simplest form.

6. (9 pts) Consider the state vector $|\psi\rangle = 4|+x\rangle + 3i|-x\rangle$.
   a. Normalize this state vector.

   b. What is the associated normalized bra, $\langle\psi| =$

   c. Write the normalized state vector in matrix form in the S_z basis.

   d. Find the probability that an SGz device will measure spin $S_z = -\frac{\hbar}{2}$.

   e. What is the probability that an SGy device will measure spin $S_y = \frac{\hbar}{2}$?

7. (4 pts) Calculate the expectation value, $\langle S_z \rangle$, and uncertainty, ΔS_z, for the wavefunction $|\psi\rangle = \frac{3}{5}|+z\rangle - \frac{4}{5}i|-z\rangle$.

8. (3 pts) It is known that there is a 25% probability of obtaining $S_x = -\frac{\hbar}{2}$ from a measurement of a spin-$\frac{1}{2}$ particle. Determine the state of the particle as completely as possible.

9. (5 pts) Apply the rotation $\hat{R}\left(-\frac{\pi}{2}\mathbf{k}\right)$ to the states and simplify:
   a. $|\psi_1\rangle = |+z\rangle$
   b. $|\psi_2\rangle = \frac{1}{\sqrt{2}}\left(|+y\rangle - i|-y\rangle\right)$

10. (4 pts) Let $|\psi\rangle = a|0\rangle + b|1\rangle$ be a two state wave function in the basis $|0\rangle, |1\rangle$, where a and b are complex numbers.
   a. Give a matrix representation of the operator $\rho = |\psi\rangle\langle\psi|$ in this basis.
   b. Give the matrix representation of the operator that projects states onto $|1\rangle$ in this basis.

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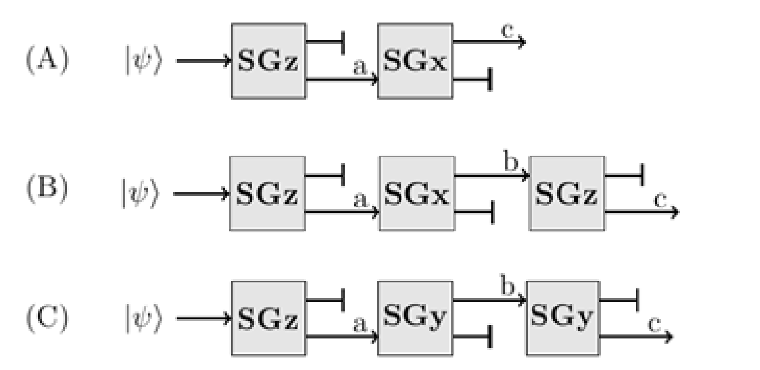
11. (6 pts) Let $\mathbb{S} = \begin{pmatrix} \langle +z|+y\rangle & \langle +z|-y\rangle \\ \langle -z|+y\rangle & \langle -z|-y\rangle \end{pmatrix}$ represent going from the S_y to the S_z basis.

a. Let $|\psi\rangle \xrightarrow[S_z]{} \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -1 \end{pmatrix}$ be the representation of a wavefunction in the S_z basis. What is the matrix representation of the wavefunction in the S_y basis?

b. $A = \frac{1}{5}\begin{pmatrix} 2 & i \\ -i & -2 \end{pmatrix}$ is a matrix representation of $\hat{A}$ in the S_z basis. What is the corresponding representation in the S_y basis?

12. (3 pts) Consider a beam of a spin-$\frac{1}{2}$ particles in state $|\psi\rangle = \frac{1}{2}|+z\rangle + \frac{\sqrt{3}}{2}|-z\rangle$. Find the percent of the particles in the beam which emerges along path c for each setup.

(A) _________ (B) _________ (C) _________



Extra Page