PHY 444 Exam I **Name** ________________________

Instructions:

- Place your name on all of the pages.
- Do all of your work in this booklet. Do not tear off any sheets.
- Show all of your steps in the problems for full credit.
- Be clear and neat in your work. Any illegible work, or scribbling in the margins, will not be graded.
- Put a box around your answers when appropriate.
- If you need more space, you may use the extra page or the back of a page and write *On back of page #* in the problem. **No other scratch paper is allowed.**

Try to answer as many problems as possible. Provide as much information as possible. Show sufficient work or rationale for full credit. Remember that some problems may require less work than brute force methods.

If you are stuck, or running out of time, indicate as completely as possible, the physics and/or steps you would take to tackle the problem. Also, indicate any relevant information that you would use.

Pace yourself – do not spend more than 15 minutes per page on your first pass.

Pay attention to the point distribution. Not all problems have the same weight.

Page	Pts	Score
1	11	
2	15	
3	14	
4	10	
Bonus		
Total	50	

Bonus – Evaluate and simplify $\hat{P}\_\hat{R}\left(\frac{\pi}{3}\mathbf{k}\right)|+y\rangle$.

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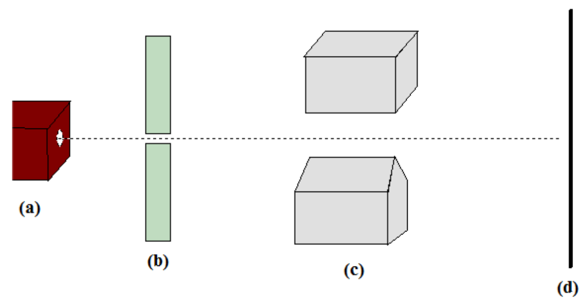
1. (3 pts) Define symbolically the following operators:
   a. Adjoint, $\hat{A}^{\dagger}$.

   b. Projection $\hat{P}_{+}$.

   c. Hermitian operator.

2. (1 pt) In the below Stern-Gerlach device a beam of silver atoms is sent through (a)-(c) and it is noticed that spin-up particles are detected in the lower portion of (d). What does that say about the poles of the magnets? (i.e., label the poles as N or S.)



3. (7 pts) A beam of silver atoms with horizontal speed of 78.51 m/s travels 1.500 m through Stern-Gerlach magnets (c) and is deflected 1.126 mm from the dotted line before leaving the magnets.

   [Note: $\mu_B = \dfrac{e\hbar}{2m_e c} = 9.274\times10^{-24}$ J/T and $M_{Ag} = 1.791\times10^{-25}$ kg.]

   a. Determine the vertical acceleration a_z of a silver atoms while in between the magnets.

   b. Use this acceleration to determine the field gradient, $\dfrac{\partial B_z}{\partial z}$, in SI units.

   c. If the atoms travel 0.500 m from (c) to (d), then when do they hit the screen?

4. (10 pts) Consider the state vector $\left|\psi\right\rangle = \left|+z\right\rangle - 2i\left|-z\right\rangle$.
   a. Normalize this state vector.

   b. What is the associated normalized bra state, $\left\langle\psi\right| =$

   c. Find the probability that an SGz device would measure spin down.

   d. Write this state vector in matrix form in the S_y basis.

   e. Find the probability that an SGy device would measure spin up for a beam of particles in state $\left|\psi\right\rangle = \left|+z\right\rangle - 2i\left|-z\right\rangle$.

5. (5 pts) Calculate the expectation value, $\left\langle S_z \right\rangle$, and the uncertainty, ΔS_z, for the wavefunction $\left|\psi\right\rangle = \frac{1}{\sqrt{10}}\left|+z\right\rangle - \frac{3i}{\sqrt{10}}\left|-z\right\rangle$.

6. (4 pts) It is known that there is a 75% probability of obtaining $S_z = -\frac{\hbar}{2}$ from a measurement of a spin-$\frac{1}{2}$ particle and the probability of finding the particle with $S_x = \frac{\hbar}{2}$ is 50%. Determine the state of the particle, $|\psi\rangle$, as completely as possible.

7. (5 pts) Apply the rotation $\hat{R}\left(\frac{\pi}{2}\mathbf{k}\right)$ to the states and simplify:

   a. $|\psi_1\rangle = |-z\rangle$

   b. $|\psi_2\rangle = \frac{1}{\sqrt{2}}\left(|+x\rangle - i|-x\rangle\right)$

8. (5 pts) Consider the matrix $A = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$. Evaluate and simplify

   a. $A^2 =$

   b. $e^{tA} =$

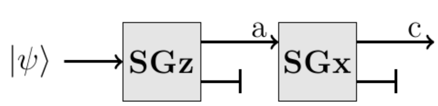
9. (7 pts) $\mathbb{S} = \begin{pmatrix} \langle +z|+x\rangle & \langle +z|-x\rangle \\ \langle -z|+x\rangle & \langle -z|-x\rangle \end{pmatrix}$ represents a transformation from an S_x to S_z basis.

   a. Let $|\psi\rangle \xrightarrow[S_z]{} \frac{1}{\sqrt{5}}\begin{pmatrix} 2 \\ -1 \end{pmatrix}$ be the representation of a wavefunction in the S_z basis. What is the matrix representation of this wavefunction in the S_x basis?

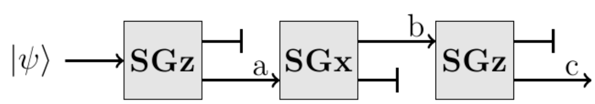
   b. Verify that the result in part a is normalized.

   c. Let $A = \begin{pmatrix} \sqrt{3} & i \\ -i & -\sqrt{3} \end{pmatrix}$ be the matrix representation of operator $\hat{A}$ in the S_z basis. What is the corresponding matrix representation in the S_x basis?

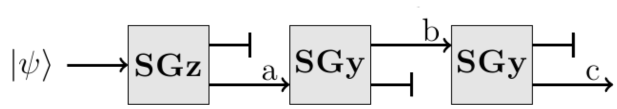
10. (3 pts) Consider a beam entering an SGz device of a $N = 2000$ spin-$\frac{1}{2}$ particles in the state $|\psi\rangle = \frac{1}{2}\left(\sqrt{3}|+z\rangle - i|-z\rangle\right)$. Find the number of the particles you expect to emerge along path c for each setup below.



(A) __________



(B) __________



(C) __________

Extra Page