

The Joy of Mathematical Puzzles and Games

Fall 2022 - R. L. Herman



Course Introduction

Syllabus and Introduction

History

Games of Chance

Famous Puzzlers

Puzzles of Smullyan

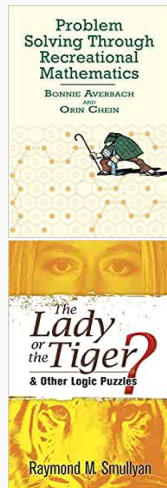
Knights and Knaves

The Lady or the Tiger

The Chess Mysteries of Sherlock

Holmes

Problem Solving Chapter 1



Syllabus

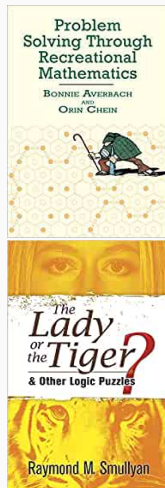
- Syllabus page
attendance, homework, exams, etc.

Texts

Materials page:

Readings, links, videos

- Schedule
- Office hours: M-F 10 AM,
Location: ST 2007J



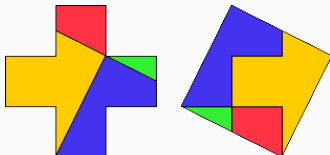
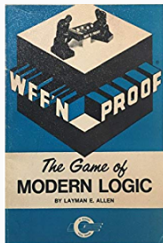
Puzzles and Games

- Logic puzzles,
- Number games,
- Geometrical puzzles,
- Games of chance
- Network problems, and
- Combinatorial problems.

How is a raven



like a writing desk?



1		31	33	63	
	51		3	19	
		49		15	
			45	61	36 13
		25			21
28	8	41	24		
43	6	55		10	
			58		38

SEND
+ MORE

MONEY

History of Games and Puzzles

- Rhind Papyrus - 1650 BCE
Long scroll 18' × 13".
Purchased 1858.
- Problem 79
- Fibonacci's *Liber Abaci*

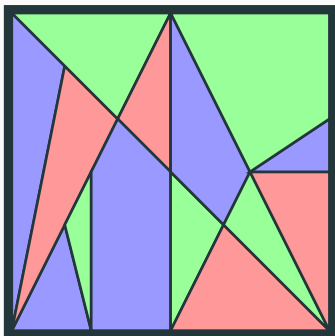
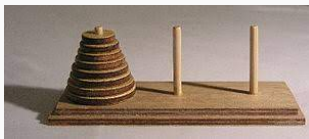
*Seven houses contain seven cats.
Each cat kills seven mice.
Each mouse had eaten seven ears of
grain.
Each ear of grain would have
produced seven hekats of wheat.
What is the total of all of these?*



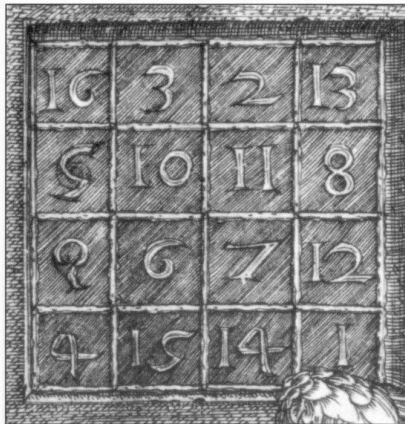
houses	7
cats	49
mice	343
spelt	2401
hekat	16807
Total	19607

Historical Puzzles

- 1256, Ibn Kallikan, total grains of wheat placed on a chess board:
18, 446, 744, 073, 709, 551, 615.
- Archimedes 287-212 BCE
 - Cattle Problem
 - Ostomachion
 - (*loculus Archimedeus*)
- Leonardo of Pisa 1100
(Fibonacci Sequence - Rabbits)
- Tower of Hanoi, Édouard Lucas, 1883.
- Magic squares
2500 BCE, Chinese, *Lo-shu*.
Dürer's *Melancholia*, 1514 - next.
- See more here.

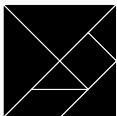


Dürer's *Melancolia*, 1514



Tangrams

- 1742, Sei Shonagon's Wisdom Plate - 7-piece - Japanese, Chie no-ita.
- 1796 - 15 piece set.
- Qi Qiao Tu, or Tangrams,
- 1st Chinese book known, 1815.
- Relation to Pythagorean Theorem.
- Chinese Tables - Banquet (1194) and Butterfly (1617).



Chinese Tangram



Japanese Tangram

HON 210 Mathematical Puzzles



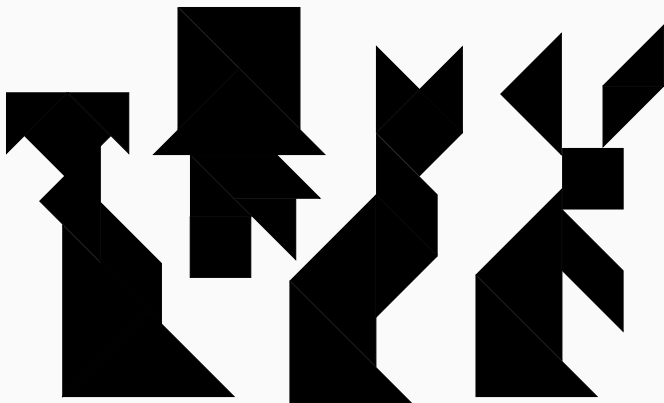
Chinese Puzzle Sets
and Tangram Table



R. L. Herman

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Alice Tangrams



Games of Chance

- Ancient China - tiles used.
'Book of Songs' refers to "the drawing of wood."
- Keno slips, 200 BCE, lottery.
- The first playing cards appeared in China in the 9th century.
- Dice evolved from 6-sided bones, astragalus, in foot.
- Egyptian - Hounds and Jackals.
- Sophocles, 500 BCE, dice were invented by a mythological hero during the siege of Troy, first mention in Greek history.
- Gambling forbidden within Rome, penalty of four times the stake being bet.
- Roman citizens invented the first gambling chips.
- Third Crusade (1190). Only knights and clergy could gamble.



The Ten Commandments, 1956



Best Board Games of Ancient World

The Problem of Points

- 1494, Luca Paccioli, *Summa de Arithmetica, Geometrica, Proportioni et Proportionalita*.
- Tackled by Cardano, Tartaglia, Galileo, but incorrect.
- 1654, Antoine Gombaud Chevalier de Méré, French essayist and gambler.
- de Méré wrote Pascal, summer.
- Pascal and Fermat corresponded July.
- Solved with different methods.
- Birth of probability theory, followed by books of Christian Huygens (1657) and Jacob Bernoulli (1713).



Two gamblers agree to toss a fair coin and each bets the same amount. They agree that the first player to win three tosses gets the stakes. The game commences:

The first player wins twice and the second player wins once.

The game ends abruptly.

de Méré asked, how do you split the pot?



Charles L. Dodgson (1832-1898)

- Mathematical logician, Author.
- Christ Church College Oxford.
First Class honours in math.
Mathematics lecturer.
- Works - Euclid, Determinants, Logic, Elections.
- Photography
- Alice Liddell, daughter of Dean Henry George Liddell.
- Charles Lutwidge in Latin: Carolus Ludovicus, Anglicise and reverse order - pen name Lewis Carroll.
- Published puzzles and stories.



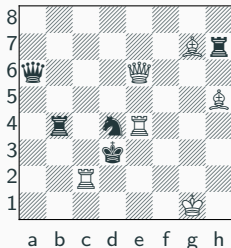
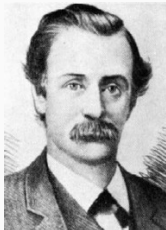
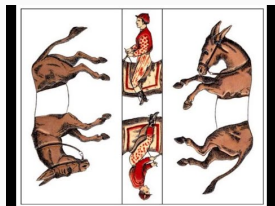
Sam Loyd (1841-1911) - "America's Greatest Puzzlist"

- 1st chess problem published: NY Saturday Courier 4/14/1855.
- Problem Ed., *Chess Monthly*, 16.
- *Chess Strategy*, 1878.
- Worked with Dudeney, scandal.
- Over 10,000 puzzles, 15 puzzle.
- Popularized tangrams.
- Sam Jr. carried on.

1	2	3	4
5	6	7	8
9	10	11	12
13	15	14	

⇒
\$1000

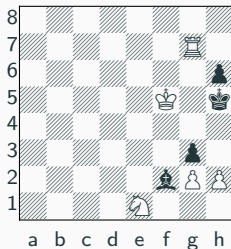
1	2	3	4
5	6	7	8
9	10	11	12
13	14	15	



White to play and mate in four.

Loyd's Most Famous Chess Problem - 1859

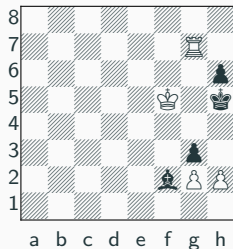
Part 1. Charles XII played with advisor Christian Albert Grosthusen when the Swedes were fighting the Turks at Bender in 1713. It's White to move and Charles announced a mate in three. [See *Chess News*, J. Fischer.]



White to play and mate in three.

Loyd's Most Famous Chess Problem - 1859

Part 2. Scarcely had he uttered the words, when a Turkish bullet, shattering the window, dashed the White knight off of the board in fragments. Grothusen started violently, but Charles, with utmost coolness, begged him to put back the other knight and work out the mate, observing that it was pretty enough. But another glance at the board made Charles smile. We do not need the knight. I can give it to you and still mate in four!

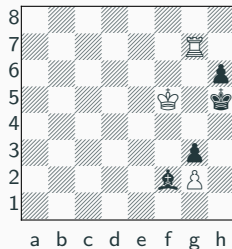


Remove white knight.

White to play and mate in four.

Loyd's Most Famous Chess Problem - 1859

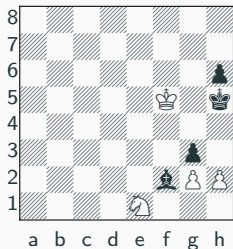
Part 3. Who would believe it, he had scarcely spoken when another bullet flew across the room, and the pawn at h2 shared the fate of the knight. Grothusen turned pale. "You have our good friends the Turks with you," said the king unconcerned, "it can scarcely be expected that I should contend against such odds; but let me see if I can dispense with that unlucky pawn. I have it!" he shouted with a tremendous laugh, "I have great pleasure in informing you that there is undoubtedly a mate in 5."



Remove white knight and white pawn at h2. White to play and mate in five.

Loyd's Most Famous Chess Problem - 1859

Yet, another part - According to Gardner - A German expert suggested that if the first bullet destroyed the white rook instead, then it would be mate in six.

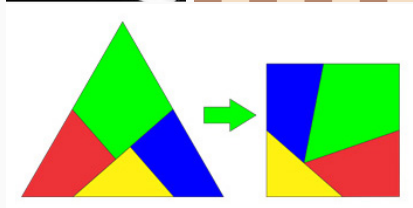
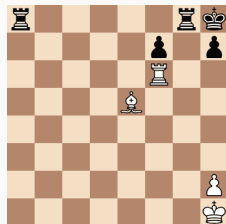
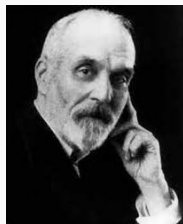


Remove white rook.

White to play and mate in six.

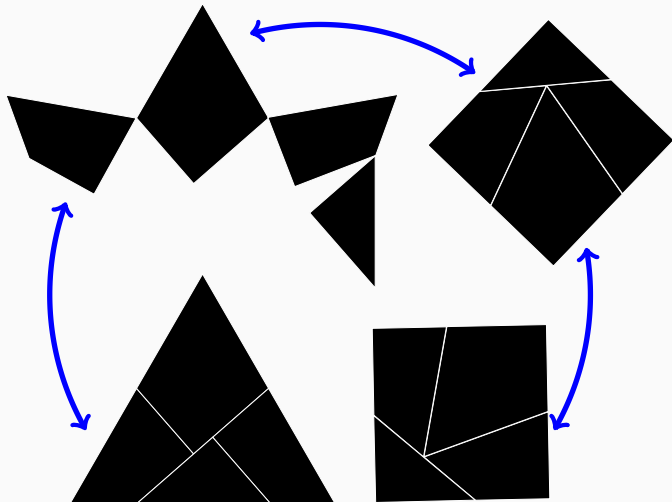
Henry Dudeney (1857-1930)

- Clerk in the Civil Service
- No college, Family of educators.
- Started with chess, puzzles.
- Pseudonym 'Sphinx'.
- 1893, corresponded with Loyd.
- Contributed to the Strand Magazine for over 30 years,
- British Chess Problem Society, Founding member, 1918.
- Puzzle collections.
- Haberdasher's problem.

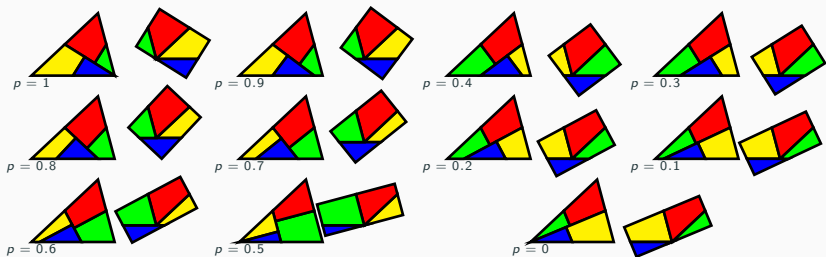


Haberdasher's Problem - Dudeney, 1902

With three cuts, dissect an equilateral triangle into a square.

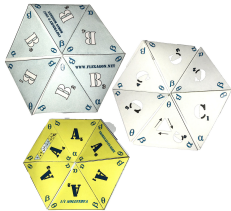
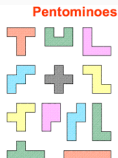


Generalizing the Haberdasher Problem



Martin Gardner (1914-2010)

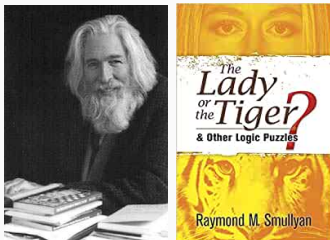
- Father, geologist, oil.
Smithsonian.
Bought Loyd's book.
- B.A., Philosophy, U. Chicago.
- Navy, radio training, destroyer.
- Graduate courses and
- Writing articles, books.
- 1956, *Scientific American*,
hexaflexagons [Arthur Stone, 1939].
'Mathematical Games' column
for 25 years.
- Over 60 books, and
numerous pamphlets.



flexagon.net

Raymond Smullyan (1919-2017)

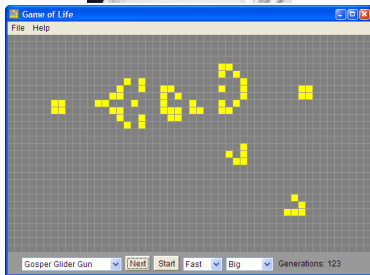
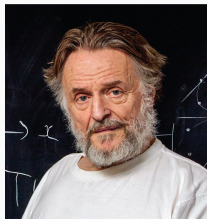
- American mathematician, magician, concert pianist, logician and philosopher.
- From Far Rockaway, NYC.
- Composed 1st chess puzzle at 16.
- Retrograde analysis.
- Unusual education.
- Piano lessons in CA.
- Studied Logic, Chicago.
- Taught -Dartmouth, 1954-1956, B.S. from Univ. of Chicago, 1955.
- Popular puzzle books emphasizing logic puzzles.



Find the missing piece
in the blue space.

John Horton Conway (1937 - 2020)

- English mathematician.
- Contributions
 - Knot theory,
 - Number theory,
 - Combinatorial games,
 - Game of Life.
- Lecturer in Pure Mathematics, University of Cambridge, 1964.
- Career took off, late 60s.
- Martin Gardner described Game of Life, Oct. 1970, *Sci. Am.*
- Shipper's Dilemma.



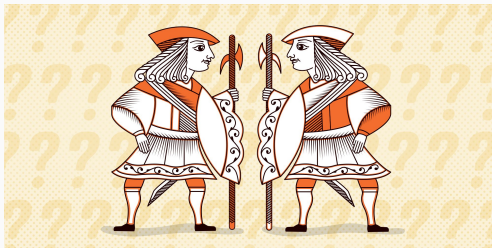
Let the Games Begin

- Ready to try some puzzles?
- From Smullyan's *The Lady or the Tiger?*
 - Knights and Knaves
 - The Lady or the Tiger
- Then, Averbach and Chen, Chapter 1.



Knights and Knaves

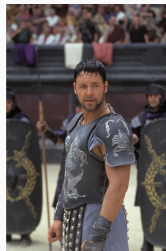
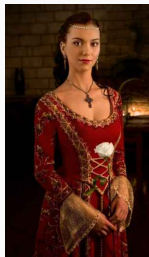
A special island is inhabited only by knights and knaves. Knights always tell the truth and knaves always lie.



You meet two inhabitants: A and B. A tells you that B is a knave. B says, “Neither A nor I are knaves.” Who is a knight and who is a knave?

The Lady or the Tiger

A King heard a story where a prisoner had to choose between two doors: behind one there was a princess, behind the other a tiger. If the prisoner chose the princess, he could marry her; if he chose the tiger, he would probably be eaten.

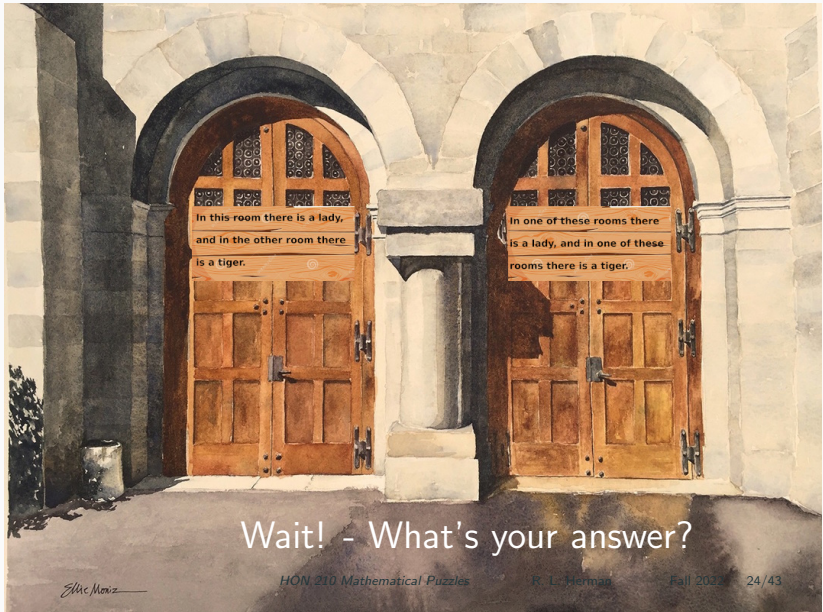


The Challenge

The King liked the idea but didn't want to leave things to chance. So, he decided to post signs on the doors giving clues as to what was behind the doors. Also, it would be possible that there are princesses behind both doors or tigers behind both doors. A clever prisoner who can reason logically would be able to save his own life.



Trial 1 - One sign is true and the other is false. Pick a door!



Trial 1 - One sign is true and the other is false. Pick a door!



Trial 1 - Solution?



E. McMoriz

Trial 1 - Informal reasoning.

Room 1 In this room there is a lady and in the other is a lion.

Room 2 In one of these rooms is a lady and in one there is a tiger.

One of the doors is true. If Door 1 is true, then Door 2 is also true.

Therefore, Door 1 is false and Door 2 is true.

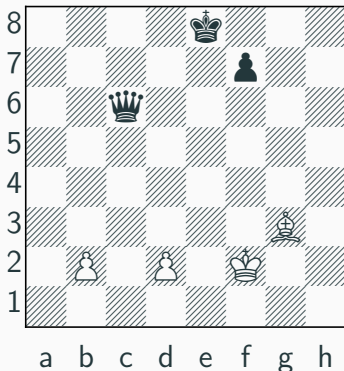
Conclusion: A tiger is behind Door 1 and a Lady behind Door 2.

Trial 2 The signs are changed. They are either both true or both false.
Which room would you pick?



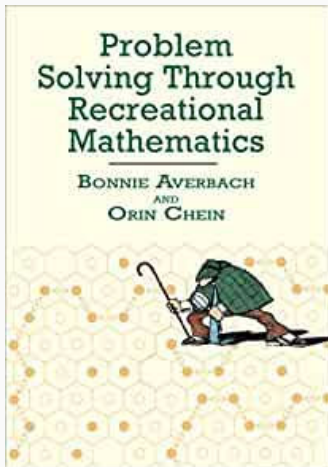
The Chess Mysteries of Sherlock Holmes

Raymond Smullyan opens his 1979 book, *The Chess Mysteries of Sherlock Holmes*, with the following chessboard position. He then asks, "Suppose I told you that in the following position no pawn has ever reached the eight square. Would you believe me?"



Problem Solving - Chapter 1

- Six Problems
- Solve using general reasoning techniques:
 - Make assumptions,
 - Look at alternatives, (reaching a contradiction)
 - Use visual aids - tables, charts, trees, diagrams.
- There is often more than one way to look at a problem.
- If you do not make progress - put it in a drawer ...



Problem 1.1

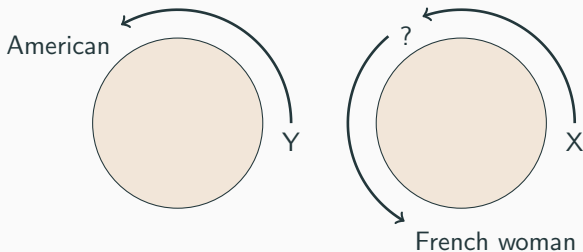
An elimination boxing tournament was organized. There were 114 participants and so there were 57 matches in the first round of the tournament. In the second round, the 57 fighters remaining were paired, resulting in 28 matches; one fighter received a bye (that is, did not have to fight in that round). The 29 fighters remaining were then paired, and so on.

- How many matches in all were required to determine a winner of the tournament?
- How many matches would be required if n people participated in the tournament (where n represents a fixed but unspecified whole number) ?

Problem 1.2

Ms. X, Ms. Y, and Ms. Z—an American woman, an Englishwoman, and a Frenchwoman, but not necessarily in that order, were seated around a circular table, playing a game of Hearts. Each passed three cards to the person on her right. Ms. Y passed three hearts to the American. Ms. X passed the queen of spades and two diamonds to the person who passed her cards to the Frenchwoman.

Who was the American? The Englishwoman? The Frenchwoman?



Problem 1.3

Armand Alloway, Basil Bennington, Col. Carlton Cunningham, Durwood Dunstan, and Everett Elmsby, Esq. are the five senior members of the Devonshire Polo Club. Each owns a pony that is named after the wife of one of the others.

Mr. Alloway's pony is named Georgette; Col. Cunningham owns Jasmine; and Mr. Elmsby owns Inez. Francine, owned by Mr. Dunstan, is named after Alloway's wife. Georgette's husband owns the pony that is named after Mr. Bennington's wife. Helene Cunningham is the only wife who knows how to ride a horse.

Who is Jasmine's husband? Who owns Helene?

Problem 1.3 - Table Method

Recall the clues:

1. Mr. Alloway's pony is Georgette.
2. Col. Cunningham owns Jasmine.
3. Mr. Elmsby owns Inez
4. Francine, owned by Mr Dunstan,
5. is named after Alloway's wife.
6. Helene Cunningham.
7. Georgette's husband owns pony named after Mr. Bennington's wife.

	F	G	H	I	J
A					
B					
C					
D					
E					

Use a Table.

Gentlemen A-E, Wives F-J.

Place ✓ if married.

Place x if not married.

Problem 1.3 - Table Method

Recall the clues:

1. Mr. Alloway's pony is Georgette.
2. Col. Cunningham owns Jasmine.
3. Mr. Elmsby owns Inez
4. Francine, owned by Mr Dunstan,
5. is named after Alloway's wife.
6. Helene Cunningham.
7. Georgette's husband owns pony named after Mr. Bennington's wife.

Use a Table.

Gentlemen A-E, Wives F-J.

Place ✓ if married.

Place x if not married.

Use Clues 5,6

	F	G	H	I	J
A	✓	x	x	x	x
B	x		x		
C	x	x	✓	x	x
D	x		x		
E	x		x		

Problem 1.3 - Table Method

Recall the clues:

1. Mr. Alloway's pony is Georgette.
2. Col. Cunningham owns Jasmine.
3. Mr. Elmsby owns Inez
4. Francine, owned by Mr Dunstan,
5. is named after Alloway's wife.
6. Helene Cunningham.
7. Georgette's husband owns pony named after Mr. Bennington's wife.

Use a Table.

Gentlemen A-E, Wives F-J.

Place ✓ if married.

Place x if not married.

Use Clues 5,6

By 3, E not married to I

	F	G	H	I	J
A	✓	x	x	x	x
B	x		x		
C	x	x	✓	x	x
D	x		x		
E	x		x	x	

Problem 1.3 - Table Method

Recall the clues:

1. Mr. Alloway's pony is Georgette.
2. Col. Cunningham owns Jasmine.
3. Mr. Elmsby owns Inez
4. Francine, owned by Mr Dunstan,
5. is named after Alloway's wife.
6. Helene Cunningham.
7. Georgette's husband owns pony named after Mr. Bennington's wife.

Use a Table.

Gentlemen A-E, Wives F-J.

Place ✓ if married.

Place x if not married.

Use Clues 5,6

By 3, E not married to I

By 7, B not married to G

	F	G	H	I	J
A	✓	x	x	x	x
B	x	x	x		
C	x	x	✓	x	x
D	x		x		
E	x		x	x	

Problem 1.3 - Table Method

Recall the clues:

1. Mr. Alloway's pony is Georgette.
2. Col. Cunningham owns Jasmine.
3. Mr. Elmsby owns Inez
4. Francine, owned by Mr Dunstan,
5. is named after Alloway's wife.
6. Helene Cunningham.
7. Georgette's husband owns pony named after Mr. Bennington's wife.

Use a Table.

Gentlemen A-E, Wives F-J.

Place ✓ if married.

Place x if not married.

Use Clues 5,6

By 3, E not married to I

By 7, B not married to G

	F	G	H	I	J
A	✓	x	x	x	x
B	x	x	x		
C	x	x	✓	x	x
D	x	✓	x	x	x
E	x	x	x	x	

Alternatives for G: - D or E.

Try D. Clues 7, 4 imply?

Problem 1.3 - Table Method

Recall the clues:

1. Mr. Alloway's pony is Georgette.
2. Col. Cunningham owns Jasmine.
3. Mr. Elmsby owns Inez
4. Francine, owned by Mr Dunstan,
5. is named after Alloway's wife.
6. Helene Cunningham.
7. Georgette's husband owns pony named after Mr. Bennington's wife.

Use a Table.

Gentlemen A-E, Wives F-J.

Place ✓ if married.

Place x if not married.

Use Clues 5,6

By 3, E not married to I

By 7, B not married to G

	F	G	H	I	J
A	✓	x	x	x	x
B	x	x	x		
C	x	x	✓	x	x
D	x	x	x		
E	x	✓	x	x	x

Alternatives for G: - D or E.

Try E. (Not D) Clue 7?

What We Learned

Alternatives

- Seek Alternatives
- Chart or Table led to solutions.
- Aided by Visualization.
- Could use different tables.
 - Horses vs. wives.
 - Men vs. horses.
- Difficult to know in advance which to use!

Presentation

Can obtain solution without writing except for checks or x's or letters in table.

To present - completed chart not convincing by itself.

People want an explanation! - the method of solution.

Provide a listing of each step, check, or x with justification.

This also is helpful to you!

Be critical before presentation (homework) so you can return and retrace your steps.

Steps in Problem Solving

- Understand given information and information sought.
What is relevant?
Restate problem.
Make a list.
Use charts, tables, diagrams, etc.
- Add logical deductions.
- Continue to add to list. deductively, until obtaining solution or impasse.
impasse = no further conclusions seem possible.
Run through again for overlooked conclusions.
- If impasse, check small number of alternatives.
Assume one true temporarily. Look for conclusions. If contradiction, conclude alternative.
Use as new fact. If no contradiction, solution not complete.
- While checking alternatives, use pencil or different color.
- Final check. Show solutions satisfy all conditions of problem.

Problem 1.4

Messrs. Baker, Dyer, Farmer, Glover, and Hosier are seated around a circular table, playing poker. Each gentleman is the namesake of the profession of one of the others. The dyer is seated two places to the left of Mr. Hosier. The baker sits two places to Mr. Baker's right. The farmer is seated to the left of Mr. Farmer. Mr Dyer is on the glover's right.

What is the name of the dyer? (H. Phillips, Problem 36)

Problem 1.4

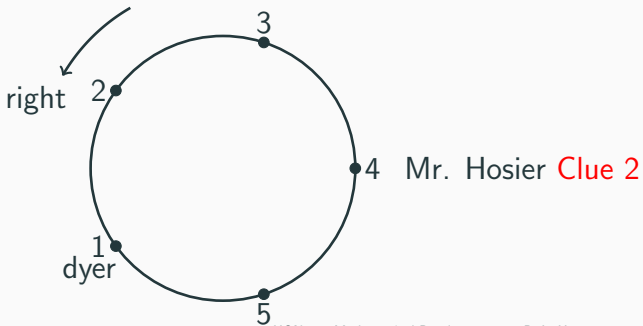
Messrs. Baker, Dyer, Farmer, Glover, and Hosier are seated around a circular table, playing poker. Each gentleman is the namesake of the profession of one of the others. The dyer is seated two places to the left of Mr. Hosier. The baker sits two places to Mr. Baker's right. The farmer is seated to the left of Mr. Farmer. Mr Dyer is on the glover's right.

What is the name of the dyer? (H. Phillips, Problem 36)

1. Each is namesake of profession of one other.
2. The dyer is seated 2 places left of Mr. Hosier.
3. The baker sits 2 places to Mr. Baker's right.
4. The farmer is on Mr. Farmer's left.
5. Mr. Dyer on glover's right.

Problem 1.4 - First Steps

1. Each is namesake of profession of one other.
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Problem 1.5

Two players, A and B, take turns in the following game. There is a pile of six matchsticks. At a turn, a player must take one or two sticks from the remaining pile. The player who takes the last stick wins. Player A makes the first move and each player always makes the best possible move.

Who wins this game?

Problem 1.6

Figure 1.1 pictures a railroad track on which are found a locomotive and two railroad cars. Car B at the right has just been filled with coal from the conveyor belt; car A at the left is empty. The tunnel is large enough to accommodate either car but not the locomotive, L. Furthermore, each car is longer than the tunnel so that, when in the tunnel, it is accessible to the locomotive from either side.

Using the locomotive to push or pull the cars, how can the two cars be made to switch places and the locomotive end up between them?

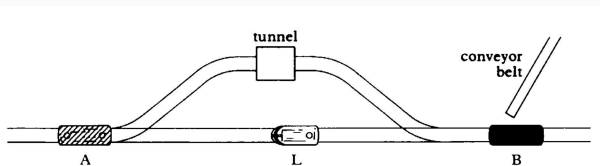


FIGURE 1.1

Solutions

Go no further!

Knights and Knaves Analysis: A and $\sim B$

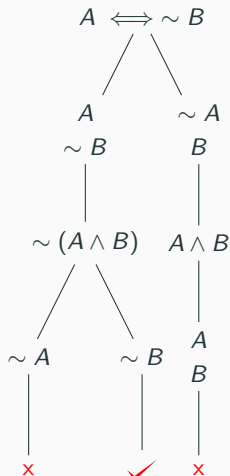
Start with the statements (A: "B is a knave." B: "Neither A nor I are knaves.") $A \iff \sim B$ and $B \iff A \wedge B$.

A	B	$\sim B$	$A \iff \sim B$	$A \wedge B$	$B \iff A \wedge B$
T	T	F	F	T	T
T	F	T	T	F	T
F	T	F	T	F	F
F	F	T	F	F	T

Table 1: Truth table.

A	B	$\sim A$	$\sim B$	$A \wedge B$	$\sim (A \wedge B)$	$\sim A \vee \sim B$
T	T	F	F	T	F	F
T	F	F	T	F	T	T
F	T	T	F	F	T	T
F	F	T	T	F	T	T

Table 2: de Morgan's Law $\sim (A \wedge B) \iff \sim A \vee \sim B$.



Trial 1 - Solution Explained with a Truth Table.

L_1 : Lion behind door 1.

P_1 : Lady behind door 1.

L_2 : Lion behind door 2.

P_2 : Lady behind door 2.

Door 1: P_1 and L_2 .

Door 2: P_1 and L_2 , or P_2 and L_1 .

L_1	L_2	P_1	P_2	$A = P_1 \wedge L_2$	$P_2 \wedge L_1$	$B = A \vee (P_2 \wedge L_1)$	$A \wedge \sim B$	$\sim A \wedge B$
T	T	T	T	T	T	T	F	F
T	T	T	F	T	F	T	F	F
T	T	F	T	F	T	T	F	T
T	T	F	F	F	F	F	F	F
T	F	T	T	F	T	T	F	T
T	F	T	F	F	F	F	F	F
T	F	F	T	F	T	T	F	T
T	F	F	F	F	F	F	F	F
F	T	T	T	T	F	T	F	F
F	T	T	F	T	F	T	F	F
F	T	F	T	F	F	F	F	F
F	T	F	F	F	F	F	F	F
F	F	T	T	F	F	F	F	F
F	F	T	F	F	F	F	F	F
F	F	F	T	F	F	F	F	F
F	F	F	F	F	F	F	F	F