Mathematics Expressions

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I. Spacetime Geodesics

We begin with the line element

$$ds^2 = g_{\alpha\beta}\, dx^\alpha dx^\beta, \quad \alpha, \beta = 0, 1, 2, 3. \tag{1}$$

We use the Variational Principle which states that freely falling test particles follow a path between two fixed points that extremizes the proper time τ, where $d\tau^2 = -\frac{ds^2}{c^2}$, or

$$c\tau_{AB} = \int_A^B \sqrt{-ds^2} = \int_A^B \sqrt{-g_{\alpha\beta}\, dx^\alpha dx^\beta}.$$

Consider a parametrized worldline, $x^\alpha = x^\alpha(\sigma)$, from point A ($\sigma = 0$) to point B ($\sigma = 1$). Then,

$$\tau_{AB} = \int_0^1 \left[-g_{\alpha\beta} \frac{dx^\alpha}{d\sigma} \frac{dx^\beta}{d\sigma} \right] d\sigma \equiv \int_0^1 L\left[x^\alpha, \frac{dx^\alpha}{d\sigma} \right] d\sigma. \tag{2}$$

Extremizing the proper time, we obtain the Euler-Lagrange equations:

$$\frac{\partial L}{\partial x^\gamma} - \frac{d}{d\sigma}\left(\frac{\partial L}{\partial (dx^\gamma/d\sigma)} \right) = 0. \tag{3}$$

Computing the needed derivatives, we find

$$\frac{\partial L}{\partial x^\gamma} = -\frac{1}{2L} \frac{\partial g_{\alpha\beta}}{\partial x^\gamma} \frac{dx^\alpha}{d\sigma} \frac{dx^\beta}{d\sigma} - \frac{L}{2} \frac{\partial g_{\alpha\beta}}{\partial x^\gamma} \frac{dx^\alpha}{d\tau} \frac{dx^\beta}{d\tau}, \tag{4}$$

$$\begin{aligned} \frac{\partial L}{\partial (dx^\gamma/d\sigma)} &= -\frac{1}{2L} g_{\alpha\beta} \left(\delta^\alpha_\gamma \frac{dx^\beta}{d\sigma} + \frac{dx^\alpha}{d\sigma} \delta^\beta_\gamma \right) \\ &= -\frac{1}{2L} \left(g_{\gamma\beta} \frac{dx^\beta}{d\sigma} + g_{\alpha\gamma} \frac{dx^\alpha}{d\sigma} \right) = -\frac{1}{L} g_{\alpha\gamma} \frac{dx^\alpha}{d\sigma}. \end{aligned} \tag{5}$$

II. Now try the following expressions:

1.

$$f(x) = \frac{2x}{(1+x^2)^2}$$

2.

$$\sqrt[3]{3\ddot{x}+7}$$

3.

$$\sum_{j=1}^{n} x_j = x_1 + x_2 + x_3 + \cdots + x_n$$

4.

$$f(x) = \left(1 + \frac{2x}{x^{21}+1}\right) - \sin x$$

5.

$$\int_0^{\pi} \sin x \, dx = 2.$$

6.

$$\begin{aligned} 2x + y &= 6 \\ x - y &= 3 \end{aligned} \tag{6}$$

7.

$$\begin{pmatrix} 1 & 2 & 3 \\ 6 & 5 & 4 \end{pmatrix}$$

8.

$$\left| 4x^3 + \left(x + \frac{42}{1+x^4}\right)^2 \right|$$

9.

$$\left.\frac{du}{dx}\right|_{x=0}$$

10.

$$|x| = \begin{cases} x & \text{if } x \geq 0; \\ -x & \text{if } x < 0. \end{cases}$$

11. Try to type these together: x_0^2, 2^{x^x}, $\lim_{n\to\infty} \frac{n!}{n^n}$ and $\sqrt{\frac{1-x}{1+x}}$

12.

$$\rho\left(\frac{\partial \mathbf{v}}{\partial t}+(\mathbf{v}\cdot\nabla)\mathbf{v}\right)=-\nabla p+\rho\mathbf{g}+\mu\nabla^2\mathbf{v}. \tag{7}$$

13.

$$\begin{aligned}(a+b)^n &= a^n\left(1+\frac{b}{a}\right)^n\\ &= a^n\left(1+n\frac{b}{a}+O\left(\left(\frac{b}{a}\right)^2\right)\right)\\ &= a^n+na^n\frac{b}{a}+a^nO\left(\left(\frac{b}{a}\right)^2\right).\end{aligned} \tag{8}$$

14.

$$-\frac{\hbar^2}{2m}\nabla^2\Psi+V\Psi=i\hbar\frac{\partial\Psi}{\partial t}$$

15.

$$x=a_0+\cfrac{1}{a_1+\cfrac{1}{a_2+\cfrac{1}{a_3+a_4}}} \tag{9}$$

16.

$$\begin{aligned}\left(1+\frac{v^2}{c^2}\right)^n &= 1+n\frac{v^2}{c^2}+\frac{n(n-1)}{2!}\left(\frac{v^2}{c^2}\right)^2\\ &\quad+\frac{n(n-1)(n-2)}{3!}\left(\frac{v^2}{c^2}\right)^3\\ &\quad+\ldots\end{aligned}$$

17.

$$\frac{d^2x^\alpha}{d\tau^2}+\Gamma^\alpha_{\delta\beta}\frac{dx^\delta}{d\tau}\frac{dx^\beta}{d\tau}=0, \tag{10}$$

$$g_{\alpha\gamma}\Gamma^\alpha_{\delta\beta}=\frac{1}{2}\left[\frac{dg_{\delta\gamma}}{dx^\beta}+\frac{dg_{\gamma\beta}}{dx^\delta}-\frac{dg_{\delta\beta}}{dx^\gamma}\right]. \tag{11}$$

18.

$$(i\partial\!\!\!/-m)\psi=0. \qquad \Box^2u=\left(\frac{1}{c^2}\frac{\partial^2}{\partial t^2}-\nabla^2\right)u=0. \tag{12}$$