Using Tikz for Spheres

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1 Simple Drawings

How can one use the **tikz** package to draw spheres? In the preamble of the LaTeX file, add

```
\usepackage{tikz}
```

and then you can create simple sketches.

In Figure 2 we draw a circle of radius 1 centered at the origin. using

```
\begin{tikzpicture}
\draw (0,0) circle(1);
\end{tikzpicture}
```

We can turn this into a sphere by adding an ellipse to the above code as seen in Figure 3.

```
\draw (0,0) ellipse (1 and .2);
```

In Figure 4 we make the ellipse a dashed line.

```
\draw[dashed] (0,0) ellipse (1 and .2);
```

A little fancier version is to make the ellipse half solid and half dashed as shown in Figure 5 using the lines

```
\draw plot[domain=pi:2*pi] ({cos(\x r)},{.2*sin(\x r)});
\draw[dashed] plot[domain=0:pi] ({cos(\x r)},{.2*sin(\x r)});
```

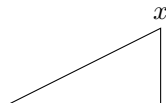The scale can be changed to make this larger using the modification
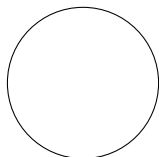

Figure 1: Caption
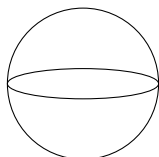
Figure 2: A simple circle of radius 1.

Figure 3: A simple sphere using a circle and an ellipse.

```
\begin{tikzpicture}[scale=2]
```

Noe, we can add axes and a vector and labels as shown in Figure 7. Further adjustments might still be desired.

```
\begin{tikzpicture}[scale=2]

% Axes
\draw[->] (0,0)--(1.2,0) node[right] {$y$};
\draw[->] (0,0)--(0,1.2) node[above] {$z$};
\draw[->] (0,0)--(-.75,-.5) node[below left] {$x$};

% Vector and label node
\draw[->] (0,0)--(.5,0.866);
\draw (.4,.5)  node {${\mathbf v}$};

\draw (0,0) circle(1);
\draw plot[domain=pi:2*pi] ({cos(\x r)},{.2*sin(\x r)});
\draw[dashed] plot[domain=0:pi] ({cos(\x r)},{.2*sin(\x r)});
\end{tikzpicture}
```

A little more work can produce the Hopf map in Figure 8. The scopes allow one to easily translate pieces with respect to one another. Here we add a sphere and plave with arrows indicating the needed maps. Further annotation can be used to enhance the figure.

```
\begin{tikzpicture}[scale=2]

% Axes
\draw[->] (0,0)--(1.2,0) node[right] {$y$};
\draw[->] (0,0)--(0,1.2) node[above] {$z$};
```
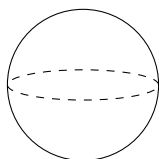
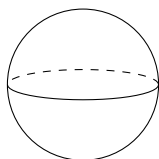Figure 4: A simple sphere using a circle and an ellipse.

Figure 5: A sphere using a circle and parametric equations for sections of an ellipse.

```
\draw[->] (0,0)--(-.75,-.5) node[below left] {$x$};

% Vector
\draw[->] (0,0)--(.5,0.866);
\draw (.4,.5)  node {${\mathbf v}$};

\draw (0,0) circle(1);
\draw plot[domain=pi:2*pi] ({cos(\x r)},{.2*sin(\x r)});
\draw[dashed] plot[domain=0:pi] ({cos(\x r)},{.2*sin(\x r)});

\begin{scope}[shift={(4,0)}]
\draw (0,0) circle(1);
\draw plot[domain=pi:2*pi] ({cos(\x r)},{.2*sin(\x r)});
\draw[dashed] plot[domain=0:pi] ({cos(\x r)},{.2*sin(\x r)});
\end{scope}

\begin{scope}[shift={(1,-2)}]
\draw (0,0)--(2,0)--(2.5,.5)--(.5,.5)--cycle;
\end{scope}

% Labels
\draw (.75,1)  node {${S^2}$};
\draw (3.2,1)  node {${S^3\subset R^4}$};
\draw[->] (3,1.1) [out=150,in=30] to (.85,1.1) ;
\draw (2,1.5)  node {Hopf Map};

\draw[->] (4,-1.1) to (3.5,-1.4) ;
\draw[->] (1.5,-1.4) to (.5,-1) ;
```
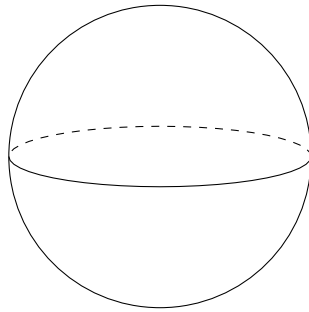
3

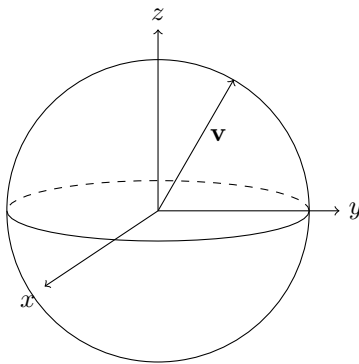Figure 6: Changing the scale by a factor of 2.

Figure 7: A sphere using a circle and parametric equations for sections of an ellipse.

```
\draw (3.9,-1.3)  node {$P$};
\draw (.9,-1.3)  node {$E$};
\end{tikzpicture}
```

Now imagine what one can do with simpler figures as in Figure 8.

```
\begin{tikzpicture}[scale=1]

\draw[->] (-2.2,0)--(2.5,0) node[right] {$C$};
\draw[->] (0,-2.2)--(0,2.5) node[above] {$C$};
\draw (0,0) circle(2);

\draw (1.75,1.75)--(-1.75,-1.75);

\end{tikzpicture}
```
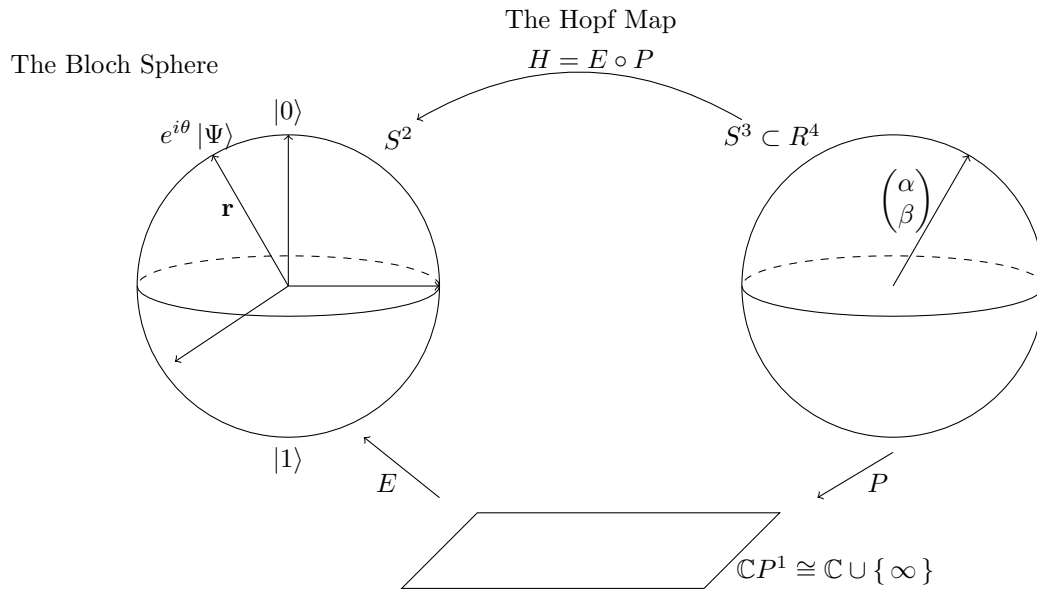
4

The Hopf Map

The Bloch Sphere

$H = E \circ P$

$e^{i\theta} \left| \Psi \right\rangle$ $\left| 0 \right\rangle$ S^2 $S^3 \subset R^4$

$\binom{\alpha}{\beta}$

r

$\left| 1 \right\rangle$ E P

$\mathbb{C}P^1 \cong \mathbb{C} \cup \{\infty\}$

Figure 8: A crude rendition of the Hopf map.

2 The Bloch Sphere Package

In the preamble of the LaTeX file, add

```
\usepackage{blochsphere}
```

to get the blochsphere package. Here are some Bloch Spheres in Figures 10-12.

3 More Bloch Sphere Examples

From StackExchange https://tex.stackexchange.com/questions/226507/pgf-draw-longitudinal-arcs-in-3d-axis-environment you get another figure using pgfplots. However, you need to load this package with

```
\makeatletter
\let\pgfmathModX=\pgfmathMod@
\usepackage{pgfplots}%
\let\pgfmathMod@=\pgfmathModX
\makeatother
```

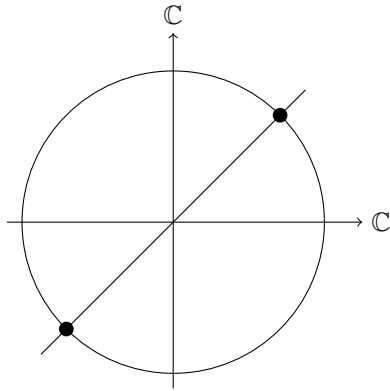in order to avoid clashing of packages. There are shown in Figures 3-3.

Figure 9: The projective space $\mathbb{C}P^1$ can be constructed by identifying the points at which taking lines in the space $\mathbb{C} \times \mathbb{C}$ intersect the unit circle in this space. Since the space is of two complex dimensions, a line is actually a plane and a circle is actually an S^3. Thus, their intersection is given by a one dimensional space which is seen to be S^1.
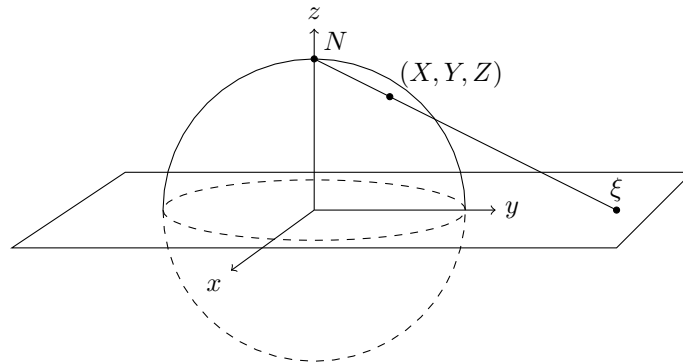
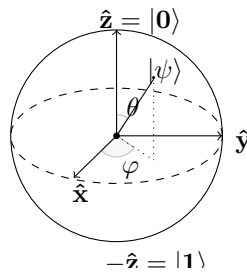

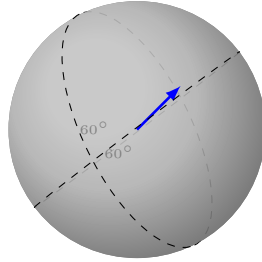Figure 10: Stereographic projection.

Figure 11: Simple version

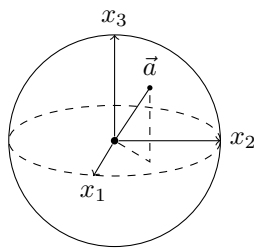Figure 12: Version using blochsphere package
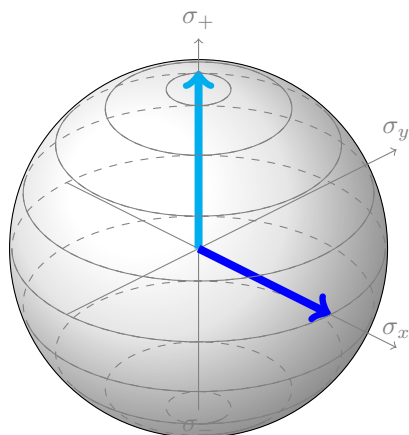

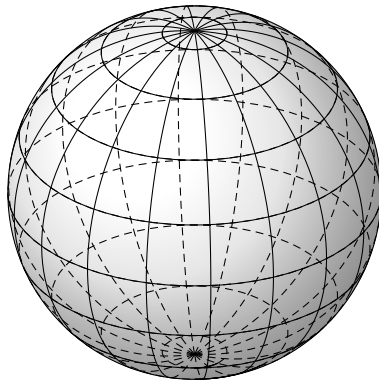Figure 13: Another simple version

Figure 14: Bloch sphere with arrows using pgfplots.

Figure 15: Bloch sphere with arrows using pgfplots.