PHY 455 – Thermal Physics – Review 3

1) Magnetic Systems
   a) Energy of interaction: $E = -\boldsymbol{\mu}\cdot\mathbf{B},\ \mu_z = s\mu,\ s = \pm 1$
   b) $Z_N = Z_1^N,\ Z_1 = \sum_{s=\pm 1} e^{s\mu\beta B} = 2\cosh(\beta\mu B).$
   c) Mean Magnetization and isothermal susceptibility, $M = \mu\sum_{i=1}^{N}\overline{s_i},\ \chi = \left(\frac{\partial M}{\partial B}\right)_T$
   d) Magnetic free energy: $dG(T,M) = -SdT + \mu_0 H dM,\ F = G - \mu_0 HM$

      $\Rightarrow \mu_0 M = -\left(\frac{\partial F}{\partial H}\right)_T,\ S = -\left(\frac{\partial F}{\partial T}\right)_H$
   e) Ising Model, 1D, $E = -J\sum_{i,j=nn(i)}^{N} s_i s_j - H\sum_{i=1}^{N} s_i$
   f) Magnetic properties
      i) Paramagnetic, diamagnetic, ferromagnetic, relation to χ
      ii) Curie point, hysteresis
2) Many Particle Systems
   a) Semiclassical limit
   b) Derive thermal de Broglie wavelength: $\lambda = \frac{h}{p_{rms}}$ and $\frac{\overline{p^2}}{2m} = \frac{3}{2}kT$, $\Rightarrow \left(\frac{2\pi\hbar^2}{mkT}\right)^{1/2}$
   c) Partition function for: particle in a box, $Z_1 = \frac{V}{\lambda^3}$, ideal gas, $Z_N = \frac{Z_1^N}{N!}$
   d) What is the entropy of mixing?
3) Classical Statistical Mechanics
   a) Equipartition Theorem – apply to simple systems
   b) Law of Dulong and Petit
   c) Maxwell velocity and speed distributions (What are differences?)
      i) $f(v)dv = 4\pi A v^2 e^{-mv^2/2kT} dv.$
      ii) Most probable speed, mean speed, rms-speed [What is rms current/voltage? Relate to peak current/voltage and average power.]
      iii) $v_{rms} = \sqrt{\frac{3kT}{m}}$
4) Occupation Numbers – $n_k =$ number of particles in single particle state with energy ε_k
   a) Canonical Ensemble $Z(V,T,N) = \sum_{\{n_k\}} e^{-\beta E_s},\quad E_s = \sum_k n_k\varepsilon_k,\quad N = \sum_k n_k.$
   b) Fermions – half-integral spin, Pauli Exclusion Principle, $n_k = 0,1.$
   c) Bosons – integral spin, $n_k = 0,1,2,\ldots$ [Be able to identify bosons and fermions.]

d) Grand Canonical Ensemble, $Z_G = \sum_s e^{-\beta(E_s - \mu N_s)}, \Omega = -kT \ln Z_G$

i) Recall: $G = F + PV = \mu N, F = E - TS, \Omega = F - \mu N$

ii) For single particle microstates: $N_s = n_k, E_s = n_k \varepsilon_k, \Rightarrow Z_{G,k} = \sum_s e^{-\beta n_k (\varepsilon_k - \mu)}$

iii) Fermions: $Z_{G,k} = 1 + e^{-\beta(\varepsilon_k - \mu)}, \Omega_k = -kT \ln Z_{G,k}$

iv) Bosons: $Z_{G,k} = \dfrac{1}{1 - e^{-\beta(\varepsilon_k - \mu)}}, \Omega_k = kT \ln[1 - e^{-\beta(\varepsilon_k - \mu)}]$

v) Distributions $\bar{n}_k = -\dfrac{\partial \Omega_k}{\partial \mu} = \dfrac{1}{e^{\beta(\varepsilon_k - \mu)} \pm 1} \quad \begin{cases} + & \text{Fermi-Dirac} \\ - & \text{Bose-Einstein} \end{cases}$

5) Single particle density of states

a) Use $g(\varepsilon)d\varepsilon = \Gamma(\varepsilon + d\varepsilon) - \Gamma(\varepsilon) \approx \dfrac{d\Gamma}{d\varepsilon} d\varepsilon$ to compute sums as integrals,

$$\sum_k f(\varepsilon_k) \to \int_0^\infty f(\varepsilon) g(\varepsilon) d\varepsilon$$

b) Use (for 3D) $\Gamma(n) = \dfrac{1}{8}\left(\dfrac{4}{3}\pi n^3\right) = \dfrac{V}{6\pi^2} k^3$ from $k = \dfrac{n\pi}{L}$ to get $g(k)dk = \dfrac{V}{2\pi^2} k^2 dk$

i) Photons: $\varepsilon = \hbar\omega = \hbar k c$

ii) Nonrelativistic particles: $\varepsilon = \dfrac{p^2}{2m}, \ p = \hbar k \Rightarrow \varepsilon = \dfrac{\hbar^2 c^2}{2m}$

iii) Relativistic particles: $\varepsilon^2 = p^2 c^2 + m^2 c^4$

6) Blackbody Radiation

a) What are Rayleigh-Jeans and Wien's laws? Why were they important historically?

b) What is Planck's distribution?

c) What is connection to CMB?

d) What is Power, Intensity, Spectral Radiance, Spectral Photon Radiance?

e) Wien's displacement law, $\dfrac{h\nu_{\max}}{kT} = 2.822, \lambda_{\max} T = 2.898 \times 10^{-3}\ \mathrm{m \cdot K}$

f) Stefan-Boltzmann law, $\dfrac{P}{A} = \sigma T^4, \ \sigma = 5.67 \times 10^{-8}\ W/m^2/K^4$

7) Named Distributions:

a) Boltzmann: $P_s = \dfrac{1}{Z} e^{-\beta E_s}$

b) Gibbs: $P_s = \dfrac{1}{Z} e^{-\beta(E_s - \mu N_s)}$

8) Partition functions

a) Canonical: $Z = \sum_s e^{-\beta E_s}$ giving $\bar{E} = -\frac{1}{Z}\frac{\partial Z}{\partial \beta}$,

b) Grand Canonical: $Z_G = \sum_s e^{-\beta(E_s - \mu N_s)}$, giving $\bar{N} = -\left(\frac{\partial \Omega}{\partial \mu}\right)_{T,V}$, $S = -\left(\frac{\partial \Omega}{\partial T}\right)_{V,\mu}$

9) Common Operations

a) $F = -kT\ln Z$, $P = \frac{1}{\beta}\frac{\partial \ln Z}{\partial V}$, $\bar{E} = -\frac{\partial \ln Z}{\partial \beta} = \frac{\partial(\beta F)}{\partial \beta}$

b) $\Omega = -kT\ln Z_G$, $P = \frac{1}{\beta}\frac{\partial \ln Z_G}{\partial V}$, $\bar{E} = -\frac{\partial \ln Z_G}{\partial \beta} + \mu N$, $N = \frac{1}{\beta}\frac{\partial \ln Z_G}{\partial \mu}$

c) $C = \frac{\partial E}{\partial T}$

d) $S = -\left(\frac{\partial F}{\partial T}\right)_{V,N}$ or use Helmholtz free energy, $F = E - TS$.

e) Continuous distributions: $P(a < x < b) = \int_a^b p(x)\,dx$, $p(x) \ge 0$ and $\int_{-\infty}^{\infty} p(x)\,dx = 1$.

f) Mean Values: $\overline{f(x)} = \sum_{i=1}^{n} f(x_i)P(i)$ or $\overline{f(x)} = \int_{-\infty}^{\infty} f(x)p(x)\,dx$.

g) Variance and standard deviation: $\mathrm{Var}(x) = \overline{x^2} - \bar{x}^2$, $\sigma_x = \sqrt{\mathrm{Var}(x)}$

h) Stirling's Formula: $\ln N! \approx N\ln N - N$, $N \gg 1$

10) Gibbs-Duhem: $E = TS - PV + \mu N$ This can be used with free energies:

a) Energy $E = E(S,V,N)$, $dE = TdS - PdV + \mu dN$.

b) Enthalpy $H(S,P,N) = E + PV$, $dH = TdS + VdP + \mu dN$

c) Helmholtz free energy $F(T,V,N) = E - TS$, $dF = -SdT - PdV + \mu dN$

d) Gibbs free energy $G(T,P,N) = F + PV$, $dG = -SdT + VdP + \mu dN$

e) Landau potential $\Omega(T,V,\mu) = F - \mu N = F - G = -PV$, $d\Omega = -SdT - PdV - Nd\mu$