1) Probability
   a) Discrete: $P(i) \geq 0, \sum_{i=1}^{N} P(i) = 1$
   b) $P(i \text{ or } j) = P(i) + P(j),\ P(i \text{ and } j) = P(i)P(j),\ P(\text{not i}) = 1 - P(i)$
   c) Continuous: $P(a < x < b) = \int_a^b p(x)\,dx,\ p(x) \geq 0$ and $\int_{-\infty}^{\infty} p(x)\,dx = 1.$
2) Mean Values, etc
   a) Mean: $\bar{x} = \sum_{i=1}^{n} x_i P(i),\ \overline{f(x)} = \sum_{i=1}^{n} f(x_i)P(i),\ \overline{x^m} = \sum_{i=1}^{n} x_i{}^m P(i),$

      $\bar{x} = \int_{-\infty}^{\infty} xp(x)\,dx,\ \overline{f(x)} = \int_{-\infty}^{\infty} f(x)p(x)\,dx,\ \overline{x^m} = \int_{-\infty}^{\infty} x^m p(x)\,dx,$
   b) Deviation: $\Delta x = x - \bar{x}$
   c) Variance: $\mathrm{Var}(x) = \overline{x^2} - \bar{x}^2$
   d) Standard Deviation: $\sigma_x = \sqrt{\mathrm{Var}(x)}$
3) Uncertainty
   a) $S(\Omega = 1) = 0,\ \Omega_1 > \Omega_2 \Rightarrow S(\Omega_1) > S(\Omega_2),\ S(\Omega_1\Omega_2) = S(\Omega_1) + S(\Omega_2)$
   b) $S = -\sum_i P_i \ln P_i \Rightarrow S(\Omega) = \ln \Omega$
   c) Principle of Maximum Uncertainty
4) Bayes' Theorem: $P(A\,|\,B) = \dfrac{P(B\,|\,A)P(A)}{P(B)},\ P(A_i\,|\,B) = \dfrac{P(B\,|\,A_i)P(A_i)}{\sum_i P(B\,|\,A_i)P(A_i)}$
5) Noninteracting Magnetic Spins $s = \pm 1,\ E = -s\mu B,\ M = \mu \sum_{i=1}^{N} s_i$
6) Binomial Distribution: $P_N(n) = \dfrac{N!}{n!(N-n)!} p^n q^{N-1} = \begin{pmatrix} N \\ n \end{pmatrix} p^n q^{N-1}$
7) Stirling's Formula: $\ln N! \approx N \ln N - N,\ N \gg 1$
8) Gaussian Distribution: $P_N(n) = \dfrac{1}{\sqrt{2\pi\sigma^2}} e^{-(n-\bar{n})/2\sigma^2}$
9) Counting Accessible Microstates, Ensembles, Equal a priori probabilities
10) Einstein Solids: $\Omega = \dfrac{(E+N-1)!}{E!(N-1)!}$,

    $\Omega_{tot} = \sum_{E_A} \Omega_A(E_A)\Omega_B(E-E_A)$, $P_A(E_A) = \dfrac{\Omega_A(E_A)\Omega_B(E-E_A)}{\Omega_{tot}}$
11) $\dfrac{1}{T} = \dfrac{\partial S}{\partial E} \approx \dfrac{S(E+\Delta E) - S(E+\Delta E)}{2\Delta E}$

12) $g(E)\Delta E = \Gamma(E+\Delta E) - \Gamma(E) \approx \frac{d\Gamma}{dE}\Delta E$

13) Particle in a box, phase space, minimum area of cell in 2D phase space.

14) Using $E_n = \frac{n^2h^2}{8mL^2}$ to compute numbers of states $\Gamma(E)$ with energy less than E for particles in boxes.

15) 1D Harmonic Oscillator: $E_n = \left(n + \frac{1}{2}\right)\hbar\omega$, and compute numbers of states $\Gamma(E)$

16) Many Noninteracting Particles:

$$\Gamma(E,V,N) = \frac{1}{N!}\left(\frac{V}{h^3}\right)^N \frac{(2\pi mE)^{3N/2}}{(3N/2)!} \text{ or } \ln\Gamma = N\ln\frac{V}{N} + \frac{3}{2}N\ln\frac{mE}{3N\pi\hbar^2} + \frac{5}{2}N$$

17) $\frac{1}{T} = \left(\frac{\partial S}{\partial E}\right)_{V,N}$, $\frac{P}{T} = \left(\frac{\partial S}{\partial V}\right)_{E,N}$, $\frac{\mu}{T} = -\left(\frac{\partial S}{\partial N}\right)_{E,V}$

18) $S = k\ln\Omega = k\ln\Gamma(E)$

19) Boltzmann distribution $P_s = \frac{1}{Z}e^{-\beta E_s}$

20) Partition function $Z = \sum_s e^{-\beta E_s}$ giving $\bar{E} = -\frac{1}{Z}\frac{\partial Z}{\partial \beta}$, $\overline{E^2} = \frac{1}{Z}\frac{\partial^2 Z}{\partial \beta^2}$

21) $C_V = \frac{\partial \bar{E}}{\partial \beta} = \frac{\mathrm{Var}(E)}{kT^2}$