Optics **Final Exam** **Name ________________________**

Instructions:

- Place your name on all of the pages.
- Do all of your work in this booklet. Do not tear off any sheets.
- Show all of your steps in the problems for full credit.
- Be clear and neat in your work. Any illegible work, or scribbling in the margins, will not be graded.
- Put a box around your answers when appropriate.
- If you need more space, you may use the back of a page and write *On back of page #* in the problem space or the attached blank sheet. **No other scratch paper is allowed.**

Try to answer as many problems as possible. Provide as much information as possible. Show sufficient work or rationale for full credit. Remember that some problems may require less work than brute force methods. **Pay attention to units and significant digits!**

If you are stuck, or running out of time, indicate as completely as possible, the methods and steps you would take to tackle the problem. Also, indicate any relevant information that you would use.

Pace yourself ! and Pay attention to the point distribution. Not all problems have the same weight.

Page	Pts	Score
1	24	
2	16	
3	24	
4	19	
5	15	
6	12	
7	15	
Total	125	

Bonus: A 20 cm diameter glass bowl with thin walls is filled with water. What is the focal length of this thick lens? At what location does the image form of a flower that is 4.0 m from this bowl?

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1. A beam of 12 cm planar microwaves is incident on the surface of a dielectric at 30^{o}. The index of refraction of the dielectric is $n = 1.4$.
   a. What is the wavelength of the waves when traveling in the dielectric?

   b. At what angle is the beam transmitted into the dielectric?

2. Describe how you could determine whether or not the light emitted by a laser pointer is linearly polarized if you have a slab of glass ($n = 1.5$) and a white wall available.

3. A linearly polarized electromagnetic plane wave is traveling in the $+x$ direction in free space. The electric field oscillates in the xy-plane with frequency of 10 MHz and amplitude 0.08 V/m.
   a. Find the period and wavelength of this wave.

   b. Write expressions for $\boldsymbol{E}$ and $\boldsymbol{B}$.

   c. Determine the intensity of the wave.

   d. In what plane does the magnetic field oscillate?

4. Light with a mean wavelength λ = 550 nm is diffracted by a circular opening of radius 3.0 mm. The diffraction pattern is viewed on a screen 0.80 m from the aperture. What is the radius of the third ring of darkness. (The first four zeros of J_1 are 3.8317, 7.0156, 10.1735, and 13.3237.)

5. A cook spills some safflower oil (*n = 1.466*) on a Plexiglas (*n = 1.51*) cutting board. The oil film appears blue (450 nm). Determine the thickness of the film.

6. Let $\psi(x,t)$ be a one dimensional wave moving with speed *v = 0.35* m/s in the negative *x* direction. At *t = 0* it has the simple Gaussian form $\psi(x,0) = Ae^{-x^2/a^2}$, where *A = 0.005* m and *a = 0.07* m. Find $\psi(-0.77\text{m}, 2.0\text{ s})$.

7. Natural light traveling in water (*n=1.33*) is incident on a glass (*n=1.52*) surface.
   a. If the reflected light is completely linearly polarized what is the angle of incidence?

   b. In which direction is the polarization of the reflected light: in, or perpendicular to, the plane of incidence?

8. A square pulse of duration τ centered at time t_0 is described by the function

$$f(t) = \begin{cases} 0, & |t - t_0| > \tau/2, \\ E_0, & |t - t_0| < \tau/2. \end{cases}$$

   a. Find its Fourier transform $F(\omega)$.

   b. Suppose the width $\Delta\omega$ of $F(\omega)$ is defined to be the smallest value of $|\omega|$ for which $F(\omega)$ vanishes. If $\tau = 2$ ns and $t_0 = 3.7$ ms, what is $\Delta\omega$?

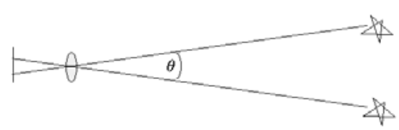
9. For silver, the complex refractive index is given by $N = 0.2 + 3.4i$.
   a. Find the distance that light of wavelength $\lambda_{vac} = 633\text{nm}$ travels inside of silver before the field is reduced by a factor of $1/e$.

   b. What is the speed of the wave crests in terms of c.

10. Light of wavelength 590 nm is normally incident on a 2.00 cm wide diffraction grating with 2000 lines per cm.
    a. At what angle will the second order peak be found?

    b. If the diffraction pattern is seen in a plane 1.50 m from the grating, then how far in cm is this spot from the central maximum?

11. Compare the angular resolutions of the Hubble Space Telescope (a 2.4 m diameter primary mirror operating with 120 nm ultraviolet radiation) and the 300 m diameter radio telescope at the Arecibo Observatory operating with 21 cm radiation.



12. Suppose a light wave that is linearly polarized in the plane of incidence impinges at 30^{o} on an air- crown glass plate (*n = 1.52*) interface.
    a. Compute the Fresnel reflection and transmission coefficients.

    b. Determine whether or not there is a phase shift in the reflected beam.

    c. What percentage of the light is transmitted?

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13. An object is placed 150 mm from a concave mirror whose radius of curvature is 600 mm.
    a. Draw the ray diagram.

    b. Explain the nature of the image (size, orientation, real or virtual).

    c. Find the distance between the image and lens.

    d. What is the magnification?

14. Consider two thin lenses with focal lengths 15.0 cm and 5.0cm and separated by 10.0 cm. Use an appropriate combination of matrices to answer the following:
    a. What is the ABCD matrix for the lens combination?

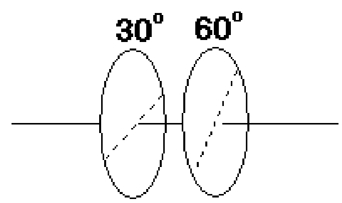
    b. What is the effective focal length of the system?

15. Let $n = \sqrt{\omega}, \tau = 0$. Find the group velocity.

16. Consider the field $\mathbf{E} = E_0\left[\mathbf{i}\sin(kz-\omega t)+\mathbf{j}\cos(kz-\omega t)\right]$ in the following.
    a. Describe the polarization state (linear, right or left circular, etc.).

    b. Write the representative Jones vector for the field in the form $\begin{bmatrix} A \\ Be^{i\delta} \end{bmatrix}$.

    c. Determine the polarization Jones matrix for the system of polarizers shown below.



    d. For the given field entering on the left, what is the intensity of the emerging beam in terms of E_0?

17. A lens of focal length +80 cm is placed 1.0 m to the left of a mirror with focal length -50.0 cm. An object is placed 1.0 m to the left of the lens.
    a. Describe the type of lens and mirror.

    b. Locate the final image formed by the light that has passed from the object, through the lens, reflected from the mirror, and passed back through the lens a second time.

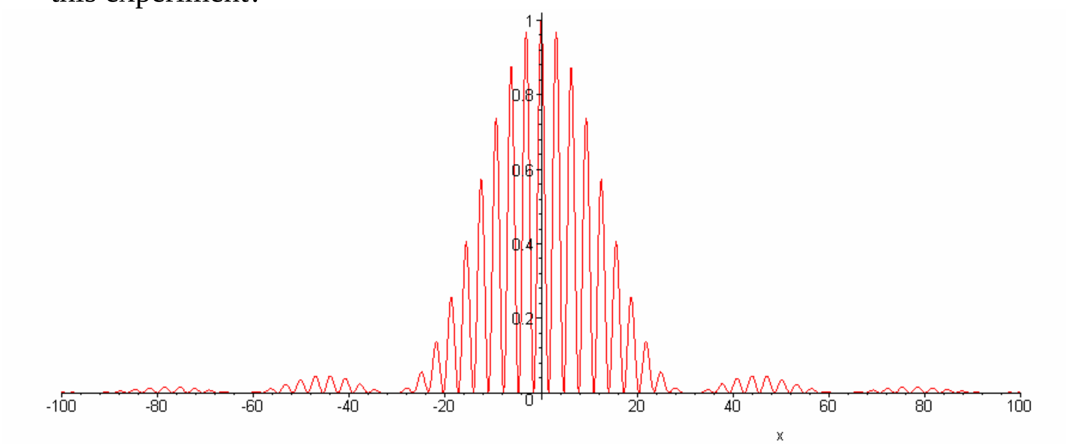
18. In a Young's double-slit experiment the slit separation is $a = 0.1$ mm and the slit width is $b = 0.01\ mm$. You put a screen 1.0 m away from the slits, and you hope to see bright and dark fringes on the screen.
    a. Someone suggests to you to use a 60W light bulb for the experiment. Why would that be silly? Does it help if you put a linear polarizer between the light bulb and the two slits?

    b. Instead of a light bulb you use a He-Ne laser emitting light with a wavelength of 633 nm. What will the distance between the dark fringes on the screen be?

    c. Explain qualitatively what diffraction (due to nonzero slit width) does to the fringe pattern.

    d. How many fringes will be visible in the central maximum?

    e. The figure shows an interference pattern from another Young's double slit experiment. What is the ratio of the slit separation *a* to the slit width *b* in this experiment?



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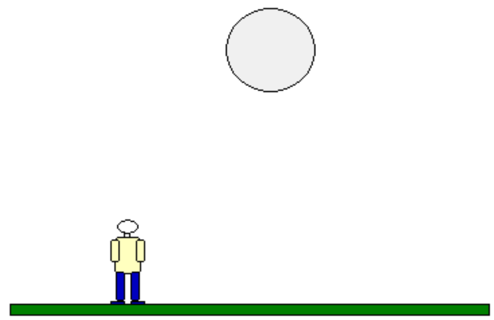
19. Answer the following about rainbows formed by the raindrop shown.



a. Draw a typical ray and the reflections which produce the observed primary rainbow. (Show the location of the sun.)
b. What is the angle at which the rainbow is found. Show it in the figure.
c. Sketch the location of the rainbow and indicate the ordering of the colors, what is seen inside and outside the rainbow.

20. The Fresnel-Kirchoff diffraction formula is given on the back page.

a. What is the obliquity factor?

b. What approximations are needed in the Fresnel approximation?

c. What approximations are used for the Fraunhofer approximation?

$\varepsilon_0 = 8.85\times10^{-12}\,\text{C}^2/\text{Nm}^2$, $\mu_0 = 4\pi\times10^{-7}\,\text{Tm/A}$,

$$r_s = \frac{n_i\cos\theta_i - n_t\cos\theta_t}{n_i\cos\theta_i + n_t\cos\theta_t},\ r_p = \frac{n_i\cos\theta_t - n_t\cos\theta_i}{n_i\cos\theta_t + n_t\cos\theta_i},$$

$$t_s = \frac{2n_i\cos\theta_i}{n_i\cos\theta_i + n_t\cos\theta_t},\ t_p = \frac{2n_i\cos\theta_t}{n_i\cos\theta_t + n_t\cos\theta_i}$$

$$E(x,y,d) = -\frac{i}{\lambda}\iint_{\text{aperture}} E(x',y',0)\frac{e^{ikR}}{R}\left[\frac{1+\cos\varphi}{2}\right]dx'dy',\quad R = \sqrt{(x-x')^2+(y-y')^2+d^2}$$

$$\frac{n_i}{d_0}+\frac{n_0}{d_i} = \frac{n_t - n_i}{r},\quad I = I_0\left[\frac{2J_1(ka\sin\theta)}{ka\sin\theta}\right]^2$$

$$R_\theta = \begin{bmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{bmatrix},\ P_h = \begin{bmatrix}1 & 0\\ 0 & 0\end{bmatrix}, P_\theta = \begin{bmatrix}\cos^2\theta & \cos\theta\sin\theta\\ \cos\theta\sin\theta & \sin^2\theta\end{bmatrix}$$

Extra Space