Optics Review II

I. Polarization

a. $\mathbf{E}(z,t) = \left(E_{0x}\hat{\mathbf{x}} + E_{0y}\hat{\mathbf{y}}\right)e^{i(kz-\omega t)} = E_{eff}\left(A\hat{\mathbf{x}} + Be^{i\delta}\hat{\mathbf{y}}\right)e^{i(kz-\omega t)}$

b. $I = \langle S\rangle_t = \frac{1}{2}nc\varepsilon_0\left|E_{eff}\right|^2$

c. Jones Vectors - $\begin{bmatrix} A \\ Be^{i\delta} \end{bmatrix} = \begin{bmatrix} \cos\alpha \\ \sin\alpha \end{bmatrix}, \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ -i \end{bmatrix}$

d. Horizontal, vertical, linear, circular, and elliptical polarizations – Write typical real/complex representations. Helicity – right and left polarized.

e. Examples (Real Form): $\left[\mathbf{i}E_{0_x} + \mathbf{j}E_{0y}\right]\cos(kz-\omega t)$, $\mathbf{E} = E_0\left[\mathbf{i}\cos(kz-\omega t) \pm \mathbf{j}\sin(kz-\omega t)\right]$,

$\mathbf{E} = \mathbf{i}\cos(kz-\omega t)E_{0_x} + \mathbf{j}\sin(kz-\omega t)E_{0y}$

f. Jones Matrices – polarizers:

i. Horizontal, Arbitrary Angle

ii. $P_h = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, R_\theta = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix},$

$P_\theta = R_\theta^{-1}P_hR_\theta = \begin{bmatrix} \cos^2\theta & \cos\theta\sin\theta \\ \cos\theta\sin\theta & \sin^2\theta \end{bmatrix}$

g. Two Polarizers:

i. $I = \frac{1}{2}nc\varepsilon_0\left|E_{eff}\right|^2\begin{bmatrix} A & Be^{i\delta} \end{bmatrix}^*\begin{bmatrix} A \\ Be^{i\delta} \end{bmatrix}$

$= \frac{1}{2}nc\varepsilon_0\left|E_{eff}\right|^2\left(|A|^2 + |B|^2\right)$

ii. Linearly Polarized: $I = I_0\cos^2(\alpha-\theta)$ (Malus' Law)

iii. Circularly Polarized & Unpolarized $I = \frac{1}{2}I_0$

iv. Three polarizers.

II. Propagation in Crystals

a. $\mathbf{P} = \varepsilon_0\chi_xE_x\hat{\mathbf{x}} + \varepsilon_0\chi_yE_y\hat{\mathbf{y}} + \varepsilon_0\chi_zE_z\hat{\mathbf{z}}$

b. $n_x = \sqrt{1+\chi_x}$, etc

c. Birefringence, ordinary-extraordinary waves, optic axis

III. Parallel Interfaces

a. $T^{tot} = \frac{T^{\max}}{1+F\sin^2\frac{\Phi}{2}}$, $T^{\max} = \frac{T^{0\to1}T^{1\to2}}{\left(1-\sqrt{R^{1\to0}R^{1\to2}}\right)^2}$, $F = \frac{4\sqrt{R^{1\to0}R^{1\to2}}}{\left(1-\sqrt{R^{1\to0}R^{1\to2}}\right)^2}$

b. Fabry-Perot etalon/interferometer, $\Phi = \frac{4\pi n_1 d}{\lambda_{vac}}\cos\theta_1 + \delta_r$

IV. Superposition and Fourier Analysis

a. Group Velocity vs Phase velocity $v_p = \frac{\omega}{k}$, $v_g = \frac{\Delta\omega}{\Delta k} \approx \frac{d\omega}{dk}$

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b. Dispersion Relation $\omega = \omega(k)$, For waves in media, $k = \frac{\omega n(\omega)}{c}$

c. Behavior of Wave Pulses - Real vs Complex, Temporal vs Frequency

d. Fourier Series $f(t) = \sum_{n=-\infty}^{\infty} c_n e^{-in\Delta\omega t}$, $c_n = \frac{\Delta\omega}{2\pi} \int_{-\pi/\Delta\omega}^{\pi/\Delta\omega} f(t) e^{in\Delta\omega t} dt$

e. Fourier Transform $\hat{f}(\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(t) e^{i\omega t} dt$, $f(t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \hat{f}(\omega) e^{-i\omega t} d\omega$

V. Interference

a. Path Difference and Phase Difference

b. Constructive/Destructive Interference -

c. Thin Films: $\delta = \frac{4\pi d n_2}{\lambda_{air}} +$ possible phase shift on reflection

or, $2d =$ phase shifts $\{0$ or $\frac{\lambda_{film}}{2}\} + \begin{cases} m\lambda_{film}, & \text{constructive} \\ \left(m + \frac{1}{2}\right)\lambda_{film}, & \text{destructive} \end{cases}$, $m = 0, 1, \ldots$

$r = \frac{n_1 - n_2}{n_1 + n_2}$ (normal incidence) => % reflected and % transmitted per interface

d. Wedges, Newton's Rings

e. Double Slits, Intensity/Irradiance

i. $d \sin\theta = n\lambda$, $n = 0, \pm 1, \ldots$ (maxima)

ii. $d \sin\theta = (2n-1)\lambda/2$, $n = 0, \pm 1, \ldots$ (minima)

iii. Locations: $y_n = L_0 \tan\theta$

iv. $\mathbf{E} = \mathbf{E}_0 \cos\left(\omega t - \frac{\delta}{2}\right) \cos\frac{\delta}{2}$ $\langle E^2 \rangle_t \sim \cos^2 \frac{\delta}{2}$

VI. Rainbows

a. Primary, Secondary Rainbows

b. Minimum Deviation, Location as function of *n*.

c. Lewin's famous questions – and answers!