Optics Final Review

I. 1D Wave Motion

a. Traveling waves: $\psi(x,t) = f(x \pm vt)$

b. Wave equation: $\psi_{tt} = v^2 \psi_{xx}$

c. Harmonic Waves: $\psi(x,t) = A \sin k(x - vt)$

d. period, wavelength, frequency, wave number, phase velocity, wave speed

e. Complex Representation: $\psi(x,t) = Ae^{i\varphi}$

II. Maxwell's Equations

a. Laws – Coulomb, Gauss, Faraday, Ampere, Maxwell-Ampere, Biot-Savart

b. Differential and Integral Forms

Differential Form	Integral Form
$\nabla \cdot \mathbf{E} = \frac{\rho}{\varepsilon_0}$	$\oint_S \mathbf{E} \cdot \mathbf{n}\, da = \frac{q_{\text{enc}}}{\varepsilon_0}$
$\nabla \cdot \mathbf{B} = 0$	$\oint_S \mathbf{B} \cdot \mathbf{n}\, da = 0$
$\nabla \times \mathbf{E} = -\frac{d\mathbf{B}}{dt}$	$\oint_C \mathbf{E} \cdot d\mathbf{l} = -\frac{d}{dt}\int_S \mathbf{B} \cdot \mathbf{n}\, da$
$\nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \mu_0 \varepsilon_0 \frac{d\mathbf{E}}{dt}$	$\oint_C \mathbf{B} \cdot d\mathbf{l} = \mu_0 I_{\text{enc}} + \mu_0 \varepsilon_0 \frac{d}{dt}\int_S \mathbf{E} \cdot \mathbf{n}\, da$

c. Wave equation: $\nabla^2 E - \mu_0 \varepsilon_0 \frac{\partial^2 E}{\partial t^2} = 0$,

d. Polarization: $\mathbf{J} = \mathbf{J}_f + \mathbf{J}_{\text{p}} = \mathbf{J}_f + \frac{\partial \mathbf{P}}{\partial t}$ and $\rho = \rho_f + \rho_p = \rho_f - \nabla \cdot \mathbf{P}$.

III. Plane Wave Solutions: $\mathbf{E}(\mathbf{r},t) = \mathbf{E}_0 \cos(\mathbf{k} \cdot \mathbf{r} - \omega t)$

a. Relations between **E** and **B**

b. Waves in Dielectrics, Susceptibility $\mathcal{N} = n + i\kappa = \sqrt{1+\chi}$

c. Index of Refraction, Absorption, Wavelength, Wave speed

d. Poynting Vector: $S = \frac{1}{\mu_0} \mathbf{E} \times \mathbf{B}$

e. Irradiance and Intensity: $\langle \mathbf{S} \rangle_t = \frac{1}{2} n \varepsilon_0 c \mathbf{E}_0 \cdot \mathbf{E}_0^* \hat{\mathbf{u}} \equiv I \hat{\mathbf{u}}$

IV. Reflection and Refraction of Plane Waves

a. Boundary Conditions: Parallel field components are continuous

b. Angle Relations: $\theta_r = \theta_i$, $n_i \sin\theta_i = n_t \sin\theta_t$

c. Brewster's Angle: $\theta_i + \theta_B = \frac{\pi}{2}$, $\theta_B = \tan^{-1} \frac{n_t}{n_i}$

d. Total Internal Reflection: $\sin\theta_c = \frac{n_t}{n_i}$

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V. Polarization

a. $\mathbf{E}(z,t) = \left(E_{0x}\hat{\mathbf{x}} + E_{0y}\hat{\mathbf{y}}\right)e^{i(kz-\omega t)} = E_{eff}\left(A\hat{\mathbf{x}} + Be^{i\delta}\hat{\mathbf{y}}\right)e^{i(kz-\omega t)}$

b. $I = \langle S \rangle_t = \frac{1}{2}nc\varepsilon_0 \left|E_{eff}\right|^2$

c. Jones Vectors - $\begin{bmatrix} A \\ Be^{i\delta} \end{bmatrix} = \begin{bmatrix} \cos\alpha \\ \sin\alpha \end{bmatrix}, \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ -i \end{bmatrix}$

d. Horizontal, vertical, linear, circular, and elliptical polarizations – Write typical real/complex representations. Helicity – right and left polarized.

e. Examples (Real Form): $\left[\mathbf{i}E_{0_x} + \mathbf{j}E_{0y}\right]\cos(kz-\omega t)$, $\mathbf{E} = E_0\left[\mathbf{i}\cos(kz-\omega t) \pm \mathbf{j}\sin(kz-\omega t)\right]$,

$\mathbf{E} = \mathbf{i}\cos(kz-\omega t)E_{0_x} + \mathbf{j}\sin(kz-\omega t)E_{0y}$

f. Jones Matrices – polarizers:

i. Horizontal, Arbitrary Angle

ii.

$$P_h = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, R_\theta = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix},$$

$$P_\theta = R_\theta^{-1}P_hR_\theta = \begin{bmatrix} \cos^2\theta & \cos\theta\sin\theta \\ \cos\theta\sin\theta & \sin^2\theta \end{bmatrix}$$

g. Two Polarizers:

i.

$$I = \frac{1}{2}nc\varepsilon_0\left|E_{eff}\right|^2\begin{bmatrix} A & Be^{i\delta} \end{bmatrix}^*\begin{bmatrix} A \\ Be^{i\delta} \end{bmatrix}$$

$$= \frac{1}{2}nc\varepsilon_0\left|E_{eff}\right|^2\left(\left|A\right|^2 + \left|B\right|^2\right)$$

ii. Linearly Polarized: $I = I_0\cos^2(\alpha-\theta)$ (Malus' Law)

iii. Circularly Polarized & Unpolarized $I = \frac{1}{2}I_0$

iv. Three polarizers.

h. Wave plates: Quarter ($\xi = e^{i\pi/2}$) and Half ($\xi = e^{i\pi} = -1$)

VI. Propagation in Crystals

a. $\mathbf{P} = \varepsilon_0\chi_xE_x\hat{\mathbf{x}} + \varepsilon_0\chi_yE_y\hat{\mathbf{y}} + \varepsilon_0\chi_zE_z\hat{\mathbf{z}}$

b. $n_x = \sqrt{1+\chi_x}$, etc

c. Birefringence, ordinary-extraordinary waves, optic axis

VII. Parallel Interfaces

a. $T^{tot} = \dfrac{T^{\max}}{1+F\sin^2\dfrac{\Phi}{2}}$, $T^{\max} = \dfrac{T^{0\to1}T^{1\to2}}{\left(1-\sqrt{R^{1\to0}R^{1\to2}}\right)^2}$, $F = \dfrac{4\sqrt{R^{1\to0}R^{1\to2}}}{\left(1-\sqrt{R^{1\to0}R^{1\to2}}\right)^2}$

b. Fabry-Perot etalon/interferometer, $\Phi = \dfrac{4\pi n_1 d}{\lambda_{vac}}\cos\theta_1 + \delta_r$

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VIII. Superposition and Fourier Analysis

a. Group Velocity vs. Phase velocity $v_p = \frac{\omega}{k}$, $v_g = \frac{\Delta\omega}{\Delta k} \approx \frac{d\omega}{dk}$

b. Dispersion Relation $\omega = \omega(k)$, For waves in media, $k = \frac{\omega n(\omega)}{c}$

c. Fourier Series $f(t) = \sum_{n=-\infty}^{\infty} c_n e^{-in\Delta\omega t}$, $c_n = \frac{\Delta\omega}{2\pi} \int_{-\pi/\Delta\omega}^{\pi/\Delta\omega} f(t) e^{in\Delta\omega t} dt$

d. Fourier Transform $\hat{f}(\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(t) e^{i\omega t} dt$, $f(t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \hat{f}(\omega) e^{-i\omega t} d\omega$

IX. Geometric Optics and Imaging

a. Fermat's Principle

b. Planes Mirrors – location/size of images in single and multiple mirror systems

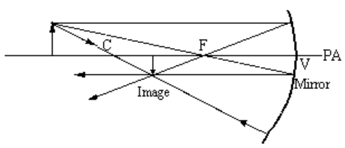
c. Spherical Mirrors

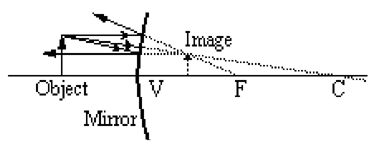
i. Concave (*R>0*), Convex (*R<0*), $f = R/2$

ii. Mirror Equation, $\frac{1}{d_0} + \frac{1}{d_i} = \frac{1}{f}$

iii. Magnification, $M = \frac{y_i}{y_0} = -\frac{d_i}{d_0}$

iv. Ray Diagrams –





d. Refracting Surfaces: $\frac{n_i}{d_0} + \frac{n_0}{d_i} = \frac{n_t - n_i}{r}$

e. Planar Surface ($r \to \infty$) - Apparent Depth

f. Thin Lenses

i. Lens Equation, $\frac{1}{d_0} + \frac{1}{d_i} = \frac{1}{f}$

ii. Lensmaker Formula, $\frac{1}{f} = (n-1)\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$,

First Surface: $r_1 > 0$ (convex), $r_1 < 0$ (concave)

Second Surface: $r_1 > 0$ (concave), $r_1 < 0$ (convex)

iii. Lens Types: Converging ($f > 0$, Positive)

Diverging ($f < 0$, Negative)

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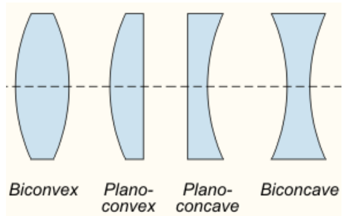
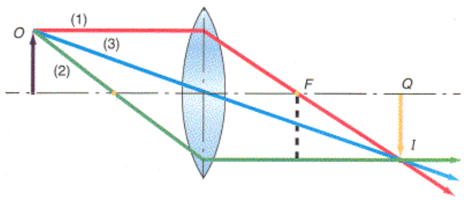
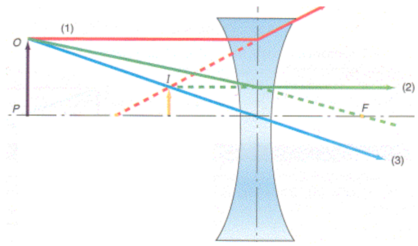


iv. Magnification, $M = \frac{y_i}{y_0} = -\frac{d_i}{d_0}$

v. Ray Diagrams





g. Matrix Formulation (Paraxial Rays)

i. General Theory: $\begin{bmatrix} y_2 \\ \theta_2 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} y_1 \\ \theta_1 \end{bmatrix}$,

ii. Imaging for $B = 0$. Magnification, $M = A$.

iii. Key Matrices:

1. Translation $\begin{bmatrix} 1 & d \\ 0 & 1 \end{bmatrix}$
2. Reflection $\begin{bmatrix} 1 & 0 \\ -\frac{2}{R} & 1 \end{bmatrix}$
3. Refraction $\begin{bmatrix} 1 & 0 \\ \frac{1}{R}\left(\frac{n_i}{n_t} - 1\right) & \frac{n_i}{n_t} \end{bmatrix}$
4. Thin Lens $\begin{bmatrix} 1 & 0 \\ -\frac{1}{f} & 1 \end{bmatrix}$

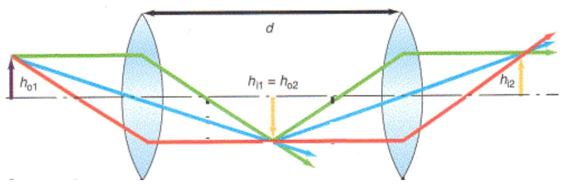
h. Complex Systems:

$$\begin{bmatrix} A' & B' \\ C' & D' \end{bmatrix} = \begin{bmatrix} 1 & d_i \\ 0 & 1 \end{bmatrix} \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} 1 & d_o \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} A + d_i C & d_o A + B + d_o d_i C + d_i D \\ C & d_o C + D \end{bmatrix}$$

$$B' = 0 \Rightarrow d_i = -\frac{B + d_o A}{D + d_o C}, \; M = A + d_i C$$

i. Example: Two Separated Lenses



$$\begin{bmatrix} A' & B' \\ C' & D' \end{bmatrix} = \begin{bmatrix} 1 & d_i \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -\frac{1}{f_2} & 1 \end{bmatrix} \begin{bmatrix} 1 & d \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -\frac{1}{f_1} & 1 \end{bmatrix} \begin{bmatrix} 1 & d_0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & d_i \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 - \frac{d}{f_1} & d \\ -\frac{1}{f_1} - \frac{1}{f_2} + \frac{d}{f_1 f_2} & 1 - \frac{d}{f_2} \end{bmatrix} \begin{bmatrix} 1 & d_0 \\ 0 & 1 \end{bmatrix}$$

X. Interference

a. Path Difference and Phase Difference, $\frac{\Delta L}{\lambda} = \frac{\Delta \phi}{2\pi}$

b. Constructive/Destructive Interference

c. Thin Films: $\delta = \frac{4\pi d n_2}{\lambda_{air}} +$ possible phase shift on reflection

or, $2d = \text{phase shifts } \{0 \text{ or } \frac{\lambda_{film}}{2}\} + \begin{cases} m\lambda_{film}, & \text{constructive} \\ \left(m + \frac{1}{2}\right)\lambda_{film}, & \text{destructive} \end{cases}, \; m = 0, 1, \ldots$

$r = \frac{n_1 - n_2}{n_1 + n_2}$ (normal incidence) => % reflected and % transmitted per interface

d. Double Slits, Intensity/Irradiance

i. $d \sin\theta = n\lambda, \; n = 0, \pm 1, \ldots$ (maxima)

ii. $d \sin\theta = (2n+1)\lambda / 2, \; n = 0, \pm 1, \ldots$ (minima)

iii. Locations: $y_n = L_0 \tan\theta$

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iv. $\mathbf{E} = \mathbf{E}_0 \cos\left(\omega t - \frac{\delta}{2}\right)\cos\frac{\delta}{2}$, gives intensity $I \propto \left\langle E^2 \right\rangle_t \propto \cos^2\frac{\delta}{2}$

XI. Diffraction

a. Huygen's Principle, Babinet's Principle (obstacle diffraction = - hole diffraction)

b. Spherical Waves, Helmholtz Equation

c. Fresnel-Kirchoff Formula: $E(x,y,d) = -\frac{i}{\lambda}\iint_{\text{aperture}} E(x',y',0)\frac{e^{ikR}}{R}\left[\frac{1+\cos\varphi}{2}\right]dx'dy'$

d. Fresnel Approximation, Fraunhofer Approximation – what are the conditions?

e. Single Slit Diffraction: $I = I_0 \text{sinc}^2\alpha,\ \alpha = \frac{\pi a}{\lambda}\sin\theta$, minima: $a\sin\theta = m\lambda,\ m = 1,2,3,...$

f. Double slit (separation *d*, width *a*): $I = I_0\cos^2\beta\,\text{sinc}^2\alpha,\ \alpha = \frac{\pi a}{\lambda}\sin\theta, \beta = \frac{\pi d}{\lambda}\sin\theta$

g. *N* Slits: $I = I_0\text{sinc}^2\alpha\left(\frac{\sin N\beta}{\sin\beta}\right)^2$, $\beta = \frac{\pi d}{\lambda}\sin\theta, \alpha = \frac{\pi a}{\lambda}\sin\theta$ [Note $N = 2$ and $N = \text{large}$ cases.]

h. Diffraction Gratings: maxima: $d\sin\theta = n\lambda,\ n = 0, \pm 1, \ldots$

i. Circular Apertures: $I = I_0\left[\frac{a\lambda}{\sin\theta}J_1\left(\frac{2\pi a}{\lambda}\sin\theta\right)\right]^2$, 1^{st} min: $\sin\theta \approx 1.22\frac{\lambda}{D}$,

j. Rayleigh's Criterion and resolvability

XII. Rainbows

a. Primary, Secondary Rainbows

b. Minimum Deviation, Location as function of *n*.

c. Know the answers to/explanation of Dr. Lewin's questions