Maxwell's Equations

Differential Form	Integral Form	Terms	Name
$\nabla\cdot\mathbf{E}=\frac{\rho}{\varepsilon_0}$	$\oint_S \mathbf{E}\cdot\mathbf{n}\,da=\frac{q_{\text{enc}}}{\varepsilon_0}$	$\oint_S \mathbf{E}\cdot\mathbf{n}\,da$ = Electric Flux	Gauss' Law for Electric Fields
$\nabla\cdot\mathbf{B}=0$	$\oint_S \mathbf{B}\cdot\mathbf{n}\,da=0$	$\oint_S \mathbf{B}\cdot\mathbf{n}\,da$ = Magnetic Flux	Gauss' Law for Magnetic Fields
$\nabla\times\mathbf{E}=-\frac{\partial\mathbf{B}}{\partial t}$	$\oint_C \mathbf{E}\cdot d\mathbf{l}=-\frac{\partial}{\partial t}\int_S \mathbf{B}\cdot\mathbf{n}\,da$	$\oint_C \mathbf{E}\cdot d\mathbf{l}$ = Electric Circulation/Emf $\frac{\partial}{\partial t}\int_S \mathbf{B}\cdot\mathbf{n}\,da$ = Rate of Change in Magnetic Flux	Faraday's Law (Note: div curl **E** = 0.)
$\nabla\times\mathbf{B}=\mu_0\mathbf{J}+\mu_0\varepsilon_0\frac{\partial\mathbf{E}}{\partial t}$	$\oint_C \mathbf{B}\cdot d\mathbf{l}=\mu_0 I_{\text{enc}}+\mu_0\varepsilon_0\frac{\partial}{\partial t}\int_S \mathbf{E}\cdot\mathbf{n}\,da$	$\oint_C \mathbf{B}\cdot d\mathbf{l}$ = Magnetic Circulation $\frac{\partial}{\partial t}\int_S \mathbf{E}\cdot\mathbf{n}\,da$ = Rate of Change in Electric Flux	Ampere-Maxwell Law (Extra term guarantees div curl **B** = 0.)

Coulomb's Law: $\mathbf{F}=q\mathbf{E}$, $\boxed{\mathbf{E}(\mathbf{r})=\frac{1}{4\pi\varepsilon_0}\int_V \rho(\mathbf{r}')\frac{\mathbf{r}-\mathbf{r}'}{|\mathbf{r}-\mathbf{r}'|^3}dv'}$ Use $\nabla_{\mathbf{r}}\cdot\frac{\mathbf{r}-\mathbf{r}'}{|\mathbf{r}-\mathbf{r}'|^3}=4\pi\delta^{(3)}(\mathbf{r}-\mathbf{r}')$ to obtain Gauss' Law

Lorentz Force: $\mathbf{F}=q\mathbf{v}\times\mathbf{B}$, Use $\nabla_{\mathbf{r}}\times\left(\mathbf{J}(\mathbf{r}')\times\frac{\mathbf{r}-\mathbf{r}'}{|\mathbf{r}-\mathbf{r}'|^3}\right)=\mathbf{J}(\mathbf{r}')\nabla_{\mathbf{r}}\cdot\frac{\mathbf{r}-\mathbf{r}'}{|\mathbf{r}-\mathbf{r}'|^3}-\left(\mathbf{J}(\mathbf{r}')\cdot\nabla_{\mathbf{r}}\right)\frac{\mathbf{r}-\mathbf{r}'}{|\mathbf{r}-\mathbf{r}'|^3}$,

Biot-Savart Law: $\boxed{\mathbf{B}(\mathbf{r})=\frac{\mu_0}{4\pi}\int_V \mathbf{J}(\mathbf{r}')\times\frac{\mathbf{r}-\mathbf{r}'}{|\mathbf{r}-\mathbf{r}'|^3}dv',}$ and **Continuity Equation**, $\boxed{\frac{\partial\rho}{\partial t}+\nabla\cdot\mathbf{J}=0,}$ to obtain **Ampere-Maxwell Law**

The **vector identity** $\nabla\times(\nabla\times\mathbf{V})=\nabla(\nabla\cdot\mathbf{V})-\nabla^2\mathbf{V}$ and Maxwell's equations lead to the **wave equation**: $\boxed{\nabla^2 E-\mu_0\varepsilon_0\frac{\partial^2 E}{\partial t^2}=0}$.

For nonmagnetic materials add **polarization**: $\mathbf{J}=\mathbf{J}_f+\mathbf{J}_{\text{p}}=\mathbf{J}_f+\frac{\partial\mathbf{P}}{\partial t}$ and $\rho=\rho_f+\rho_p=\rho_f-\nabla\cdot\mathbf{P}$.

Maxwell's Equations (separating free charge ρ_f and free current $\mathbf{J}_f$ contributions):

$$\boxed{\nabla\cdot\mathbf{D}=\rho_f,\ \nabla\cdot\mathbf{B}=0,\ \nabla\times\mathbf{E}=-\frac{\partial\mathbf{B}}{\partial t},\ \nabla\times\mathbf{H}=\mathbf{J}_f+\frac{\partial\mathbf{D}}{\partial t},\ \text{where } \mathbf{D}=\varepsilon_0\mathbf{E}+\mathbf{P}=\varepsilon\mathbf{E},\ \mathbf{H}=\frac{1}{\mu_0}\mathbf{B}-\mathbf{M}=\frac{1}{\mu}\mathbf{B}\ \text{for } \mathbf{P}=\varepsilon_0\chi_e\mathbf{E}\ \text{and } \mathbf{M}=\chi_m\mathbf{H}.}$$