Diffraction Summary

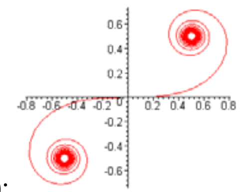
Huygen's Principle, Babinet's Principle, Spherical Waves, Helmhotz Equation: $\nabla^2 E + k^2 E = 0$ for $E(x,y,z,t) = Ee^{-i\omega t}$

Fresnel-Kirchoff Formula $E(x,y,d) = -\frac{i}{\lambda}\iint_{\text{aperture}} E(x',y',0)\frac{e^{ikR}}{R}\left[\frac{1+\cos\varphi}{2}\right]dx'dy'$

Fresnel Approximation $E(x,y,d) = -\frac{i}{\lambda d}e^{ikd}e^{i\frac{k}{2d}(x^2+y^2)}\iint_{\text{aperture}} E(x',y',0)e^{i\frac{k}{2d}(x'^2+y'^2)}e^{-i\frac{k}{d}(xx'+yy')}dx'dy' = -\frac{i}{\lambda d}e^{ikd}\iint_{\text{aperture}} E(x',y',0)e^{i\frac{k}{2d}\left[(x-x')^2+(y-y')^2\right]}dx'dy'$

Fresnel Integrals and Cornu Spirals $E(x,y,d) = -\frac{iE_0e^{ikd}}{\sqrt{2\lambda d}}\iint_{\text{aperture}} e^{i\frac{\pi}{2}(u^2+v^2)}dudv$, $\int_0^u e^{i\pi u^2/2}du = \int_0^u \cos(\frac{\pi u^2}{2})du + i\int_0^u \sin(\frac{\pi u^2}{2})du \equiv C(u) + iS(u)$

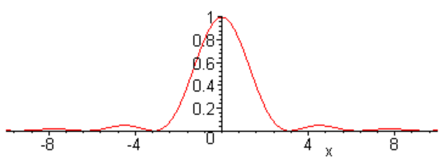
Plot S(u) vs C(u):



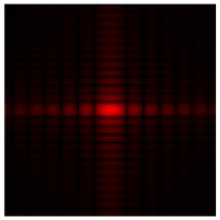
Fraunhofer Approximation $E(x,y,d) = -\frac{i}{\lambda d}e^{ikd}e^{i\frac{k}{2d}(x^2+y^2)}\iint_{\text{aperture}} E(x',y',0)e^{-i\frac{k}{d}(xx'+yy')}dx'dy'$

Diffraction Patterns

One Slit: $I = I_0\text{sinc}^2\beta,\ \beta = \frac{kb}{2}\sin\theta$



Rectangular Slit $I = I_0\text{sinc}^2\frac{kax}{2R}\text{sinc}^2\frac{kby}{2R}$,



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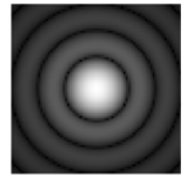
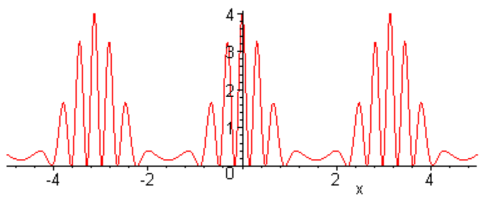


Circular Slit $E(\rho,d) = -\frac{2\pi i e^{ikd}}{\lambda d} e^{i\frac{k\rho^2}{2d}} E_0 \int_0^a J_0\left(\frac{k\rho\rho'}{d}\right)\rho' \, d\rho'$ implies $I = I_0\left[\frac{a\lambda}{\sin\theta} J_1\left(\frac{2\pi a}{\lambda}\sin\theta\right)\right]^2$

Resolution: Want $ka\sin\theta = 3.83 \Rightarrow \rho = \frac{3.83R\lambda}{2\pi a} \approx 1.22\frac{R\lambda}{2a}$

Two Slits $I = 4I_0 \operatorname{sinc}^2\beta \cos^2\alpha,\ \beta = \frac{kb}{2}\sin\theta, \alpha = \frac{ka}{2}\sin\theta$ for slit width b and separation a

- Principal vs subsidiary maxima



N Slits $I = I_0 \operatorname{sinc}^2\beta \left(\frac{\sin N\alpha}{\sin\alpha}\right)^2,\ \beta = \frac{kb}{2}\sin\theta, \alpha = \frac{ka}{2}\sin\theta$