PHY 311 Review

Things you should be comfortable with for the final:

A. General Methods

1) Separation of Variables
   Know how to carry out process; know how to write down standard solutions to various boundary value problems; $u(x,y,z) = X(x)Y(y)Z(z)$
2) Fourier Series
   Know basic formulae for Fourier series and Fourier coefficients for trigonometric, sine and cosine series and be able to sketch periodic extensions. $f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty}\left[a_n \cos\frac{n\pi x}{L} + b_n \sin\frac{n\pi x}{L}\right]$, $a_n = \frac{1}{L}\int_{-L}^{L} f(x)\cos\frac{n\pi x}{L}\,dx,$
   etc
3) Laplace Transforms
   i. Definition $Y(s) \equiv L\{y(t)\} \equiv \int_0^{\infty} y(t)e^{-st}\ dt.$
   ii. Particular Functions $t^n, e^{at}, \sin at, \cos at, H(t-a), \delta(t-a)$
   iii. Differential Equations
   $L\left\{\frac{dy}{dt}\right\} = sY(s) - y(0)$, $L\left\{\frac{d^2y}{dt^2}\right\} = s^2Y(s) - sy(0) - y'(0)$
   iv. Bromwich Integral $f(t) = \frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty} F(s)e^{st}ds$
4) Residue Theory for contour integrals $\oint f(z)\,dz = 2\pi i \sum \text{Residues}$

B. Equations

1) Wave Equation $u_{tt} = k\nabla^2 u$
2) Heat Equation $u_t = c^2\nabla^2 u$
3) Laplace's Equation $\nabla^2 u = 0$

C. Boundary Conditions

1) Fixed $u(0) = 0, \quad u(L) = 0$
2) Free or insulated $u_x(0) = 0, \quad u_x(L) = 0$
3) Periodic Boundary Conditions $u(-\pi) = u(\pi), \quad u_\theta(-\pi) = u_\theta(\pi)$

D. **Geometries in 1D-3D** [You will need to know which to apply to what problem, but non-Cartesian expressions will be given.]

1) Rectangular $\nabla^2 u = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}$
2) Polar $\nabla^2 u = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2}$
3) Cylindrical $\nabla^2 u = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2} + \frac{\partial^2 u}{\partial z^2}$
4) Spherical $\nabla^2 u = \frac{1}{\rho^2}\frac{\partial}{\partial \rho}\left(\rho^2\frac{\partial u}{\partial \rho}\right) + \frac{1}{\rho^2 \sin\phi}\frac{\partial}{\partial \phi}\left(\sin\phi\frac{\partial u}{\partial \phi}\right) + \frac{1}{\rho^2\sin^2\phi}\frac{\partial^2 u}{\partial \theta^2}$

E. Sturm-Liouville Problems

1) Put ODEs in Sturm-Liouville Form $Ly = \frac{d}{dx}\left(p\frac{dy}{dx}\right) + qy = -\lambda\sigma y.$
2) Lagrange Identity $uLv - vLu = \frac{d}{dx}[puv' - pu'v]$
3) Proof that eigenvalues are real and eigenfunctions are orthogonal

F. Vector Analysis

1) Divergence, Gradient and Curl
2) Vector Identities
3) Kronecker Delta and Levi-Civita Symbol

G. Special Solutions

1) Constant Coefficient ODEs
2) Cauchy-Euler Equations: $ax^2y'' + bxy' + cy = 0,\ y = y(x)$
   i. Solve characteristic equation $ar(r-1) + br + c = 0.$
   ii. Three Cases:
      1. $k_1x^{r_1} + k_2x^{r_2}$
      2. $x^r(k_1 + k_2 \ln|x|)$
      3. $x^\alpha[k_1\cos(\beta\ln|x|) + k_2\sin(\beta\ln|x|)]$, $r = \alpha \pm i\beta.$
3) Trigonometric and hyperbolic functions
4) Bessel Functions – General Behavior
5) Legendre Functions – General Behavior
   i. Rodrigues Formula - given
   ii. Three Term Recursion Relation - given
6) Spherical Harmonics – See the Spherical Harmonics pictures
7) Know what solutions arise for different types of problems
8) Vibration Modes of Rectangular and Circular Membranes and relation to frequencies: $2\pi f_{mn} = \omega_{mn} = c\sqrt{\lambda_{mn}}.$

   Rectangular: $\sqrt{\lambda_{mn}} = \sqrt{\left(\frac{n\pi}{L}\right)^2 + \left(\frac{m\pi}{H}\right)^2}$

   Circular: $\sqrt{\lambda_{mn}} = \frac{z_{mn}}{a}.$