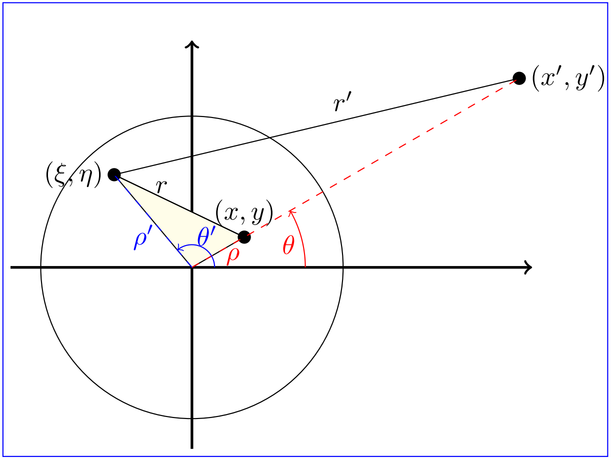

Distances

$$\rho = \sqrt{x^2 + y^2}, \quad \rho' = \sqrt{\xi^2 + \eta^2}$$

$$\sqrt{x'^2 + y'^2} = \frac{1}{\rho}$$

$$r = \sqrt{(\xi - x)^2 + (\eta - y)^2}$$

$$r' = \sqrt{(\xi - x')^2 + (\eta - y')^2}$$

Law of Cosines

$$r^2 = \rho^2 + \rho'^2 - 2\rho\rho' \cos(\theta' - \theta)$$

$$r'^2 = \frac{1}{\rho^2} + \rho'^2 - 2\frac{\rho'}{\rho} \cos(\theta' - \theta)$$

Green's Function

$$\left.\frac{r^2}{r'^2}\right|_{\rho'=1} = \frac{\rho^2 + 1 - 2\rho \cos(\theta' - \theta)}{\frac{1}{\rho^2} + 1 - \frac{2}{\rho} \cos(\theta' - \theta)} = \rho^2$$

$$G(x, y; \xi, \eta) = \frac{1}{2\pi} \ln \frac{r}{r'\rho} = \frac{1}{4\pi} \ln \frac{\rho^2 + \rho'^2 - 2\rho\rho' \cos(\theta' - \theta)}{1 + \rho^2\rho'^2 - 2\rho\rho' \cos(\theta' - \theta)}$$

Poisson Equation with Nonhomogeneous BCs

$$\nabla^2 u = f \quad \text{in } D \qquad \nabla^2 G = \delta(\xi - x, \eta - y) \quad \text{in } D$$

$$u = g \quad \text{on } C \qquad G \equiv 0 \quad \text{on } C$$

$$u(x, y) = \int_D G(x, y; \xi, \eta) f(\xi, \eta) \, d\xi d\eta + \int_C (u \nabla_{r'} G - G \nabla_{r'} u) \cdot \mathbf{n} \, ds'$$

$$\nabla_{r'} G \cdot \mathbf{n} = \frac{1}{2\pi} \frac{1 - \rho^2}{1 + \rho^2 - 2\rho \cos(\theta - \theta')}$$

$$u(x, y) = \frac{1}{4\pi} \int_0^{2\pi} \int_0^1 \ln \frac{\rho^2 + \rho'^2 - 2\rho\rho' \cos(\theta' - \theta)}{1 + \rho^2\rho'^2 - 2\rho\rho' \cos(\theta' - \theta)} f(\theta') \, \rho' d\rho' d\theta' + \frac{1}{2\pi} \int_0^{2\pi} \frac{1 - \rho^2}{1 + \rho^2 - 2\rho \cos(\theta - \theta')} g(\theta') \, d\theta'$$