Midterm Topics

I. First Order PDEs
   a. Quasilinear Equations $a(x,y,u)u_x + b(x,y,u)u_y = c(x,y,u)$
   b. Method of Characteristics $\frac{dx}{a} = \frac{dy}{b} = \frac{du}{c} = d\tau$ or $\frac{dx}{d\tau} = a, \frac{dy}{d\tau} = b, \frac{du}{d\tau} = c,$
   c. Conservation Laws
      i. Integral form: $\frac{dQ}{dt} = \phi(a,t) - \phi(b,t) + \int_a^b f(x,t)\,dx$
      ii. Local Form: $\frac{\partial u}{\partial t} + \frac{\partial \phi}{\partial x} = f.$
      iii. Constitutive Equation: $\phi = \phi(u) \Rightarrow u_t + \frac{d\phi}{du}u_x = f$
   d. Solutions of $u_t + c(u)u_x = 0.$
      i. Sketching Characteristic Curves
      ii. Sketching Solutions based on Characteristics
      iii. Breaking Time $t_B = \left\{ -\frac{1}{F'(\xi)} \right\}_{\min}$ for $F(\xi) = c(f(\xi))$ and $f(x) = u(x,0)$
      iv. Rarefactions - $u(x,t) = g(\frac{x}{t})$
      v. Shock Waves – Rankine-Hugonoit Condition $\frac{dx_s}{dt} = \frac{[\phi]}{[u]}$

         where $[u] = u^+ - u^-$
   e. General Nonlinear First Order PDEs
      i. $F(x,y,u,p,q) = 0$ for $p = u_x$, $q = u_y$
      ii. Charpit Equations $\frac{dx}{F_p} = \frac{dy}{F_q} = \frac{du}{pF_p + qF_q} = \frac{-dp}{F_x + pF_u} = \frac{-dq}{F_y + qF_u} = dt$

II. Nonhomogeneous Problems
   a. Time Independent/Time Dependent
   b. Eigenfunction Expansions
   c. Green's Formulae
   d. Examples
      i. Forced Wave Equation – Resonance
      ii. Poisson's Equation – Rectangle/Disk

III. Green's Functions
   a. ODEs
      i. Variation of Parameters
      ii. Eigenfunction Expansions
   b. Dirac Delta Function
   c. Fredholm Alternative – Generalized Green's Functions
   d. PDEs – Heat Equaton, Wave Equation, Laplace's Equation
   e. Multidimensional Problems – Poisson's Equation
      i. Infinite/Semi-infinite Problems – Method of Images
      ii. Poisson's Equation - Green's Functions