MAT 519 Final Topics

I. Complex Analysis
  a. Complex Numbers and Functions
  b. Derivatives, Cauchy-Riemann Equations, Harmonic Functions
  c. Integration along Paths
  d. Series Expansions
    i. Analytic Functions, Laurent series, Taylor series
    ii. Using geometric series
      1. $\frac{1}{1-z} = \sum_{n=0}^{\infty} z^n, |z| < 1,$
      2. $\frac{1}{1-z} = \frac{-1}{z} \times \frac{1}{1-\frac{1}{z}} = -\sum_{n=1}^{\infty} z^{-n}, |z| > 1$
  e. Singularities – removable, poles, essential
  f. Integration
    i. Cauchy's Theorem, Cauchy Integral Theorem
    ii. Computing Residues
      1. $\text{Res}[f(z); z = z_0] = \lim_{z \to z_0} (z - z_0) f(z)$ - simple poles
      2. $\text{Res}[f(z); z = z_0] = \lim_{z \to z_0} \frac{1}{k!} \frac{d^{k-1}}{dz^{k-1}} [(z - z_0)^k f(z)]$ - poles of order k
    iii. Residue Theorem $\int_C f(z)\, dz = 2\pi i \sum_{\text{Poles inside C}}$ Residues
    iv. Integrands of the form $f(\cos\theta, \sin\theta)$:
      Use $\cos\theta = \frac{z + z^{-1}}{2}$, $\sin\theta = \frac{z - z^{-1}}{2i}$
    v. Infinite (Principal Value) Integrals
    vi. Jordan's Lemma
  g. Fluid Flow
    i. Streamlines and potential functions
    ii. Types of flow – uniform, sources, sinks, vortices
  h. Conformal Mappings, Bilinear transformations, and Laplace's Equation

II. Fourier Transforms
  a. Definition $F(\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} f(x) e^{i\omega x} dx$, $f(x) = \int_{-\infty}^{\infty} F(\omega) e^{-i\omega x} d\omega$
  b. Properties – Transforms of derivatives,
  c. Gaussian Integrals, Transform of a Gaussian
  d. Convolution Theorem $F^{-1}\left[F(\omega)G(\omega)\right] = (f * g)(x) \equiv \int_{-\infty}^{\infty} f(\xi) g(x - \xi)\, d\xi$
  e. Parseval's Identity $\int_{-\infty}^{\infty} \left|F(\omega)\right|^2 d\omega = \frac{1}{2\pi} \int_{-\infty}^{\infty} \left|f(x)\right|^2 dx$
  f. Solution of PDEs using Fourier Transforms
    i. Heat Equation and the Heat Kernel

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ii. Laplace's Equation,

III. Laplace Transform

a. Definition $Y(s) \equiv L\{y(t)\} \equiv \int_0^\infty y(t)e^{-st}\,dt.$

b. Particular Functions $t^n, e^{at}, \sin at, \cos at, H(t-a), \delta(t-a), \ldots$

$f(t)$	$F(s)$	$f(t)$	$F(s)$
c	$\frac{c}{s}$	e^{at}	$\frac{1}{s-a}, s<a.$
t^n	$\frac{n!}{s^{n+1}}, s>0.$	$t^n e^{at}$	$\frac{n!}{(s-a)^{n+1}}.$
$\sin \omega t$	$\frac{\omega}{s^2+\omega^2}$	$e^{at}\sin \omega t$	$\frac{\omega}{(s-a)^2+\omega^2}$
$\cos \omega t$	$\frac{s}{s^2+\omega^2}$	$e^{at}\cos \omega t$	$\frac{s-a}{(s-a)^2+\omega^2}$
$\sinh at$	$\frac{a}{s^2-a^2}$	$\cosh at$	$\frac{s}{s^2-a^2}$
$H(t-a)$	$\frac{e^{-as}}{s}, s>0$	$\delta(t-a)$	$e^{-as},\ a\ge 0, s>0.$

c. Properties

$$L\{af(t)+bg(t)\} = aF(s)+bG(s).$$
$$L\left\{\frac{dy}{dt}\right\} = sY(s)-y(0).$$
$$L\left\{\frac{d^2y}{dt^2}\right\} = s^2Y(s)-sy(0)-y'(0).$$
$$L\left\{e^{at}y(t)\right\} = Y(s-a).$$
$$L\left\{H(t-a)y(t-a)\right\} = e^{-as}Y(s).$$
$$L\{tf(t)\} = -\frac{d}{ds}F(s).$$

d. Convolution $(f*g)(t) = \int_0^t f(t-u)g(u)\,du.$

e. Bromwich Integral $f(t) = \frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty} F(s)e^{st}\,ds$

f. Application to PDEs

IV. Numerical Solutions of PDEs

a. Finite Differences: Forward, Backward and Center differences

b. Truncation Error

c. von Neumann stability analysis and CFL

d. Matrix Methods