

Prologue

“How can it be that mathematics, being after all a product of human thought independent of experience, is so admirably adapted to the objects of reality?.” - Albert Einstein (1879-1955)

Introduction

THIS SET OF NOTES IS BEING COMPILED for use in a two semester course on mathematical methods for the solution of partial differential equations typically taken by majors in mathematics, the physical sciences, and engineering. Partial differential equations often arise in the study of problems in applied mathematics, mathematical physics, physical oceanography, meteorology, engineering, and biology, economics, and just about everything else. However, many of the key methods for studying such equations extend back to problems in physics and geometry. In this course we will investigate analytical, graphical, and approximate solutions of some standard partial differential equations. We will study the theory, methods of solution and applications of partial differential equations.

We will first introduce partial differential equations and a few models. A PDE, for short, is an equation involving the derivatives of some unknown multivariable function. It is a natural extension of ordinary differential equations (ODEs), which are differential equations for an unknown function of one variable. We will begin by classifying some of these equations.

While it is customary to begin the study of PDEs with the one dimensional heat and wave equations, we will begin with first order PDEs and then proceed to the other second order equations. This will allow for an understanding of characteristics and also open the door to the study of some nonlinear equations related to some current research in the evolution of wave equations.

There are different methods for solving partial differential and these will be explored throughout the course. As we progress through the course, we will introduce standard numerical methods since knowing how to numerically solve differential equations can be useful in research. We will also look into the standard solutions, including separation of variables, starting in one dimension and then proceeding to higher dimensions. This naturally leads to finding solutions as Fourier series and special functions, such as

Legendre polynomials and Bessel functions.

The specific topics to be studied and approximate number of lectures will include

First Semester: (26 lectures)

- Introduction (1)
- First Order PDEs (2)
- Traveling Waves (1)
- Shock and Rarefaction Waves (2)
- Second Order PDEs (1)
- 1D Heat Equation (1)
- 1D Wave Equation - d'Alembert Solution (2)
- Separation of Variables (1)
- Fourier Series (4)
- Equations in 2D - Laplace's Equation, Vibrating Membranes (4)
- Numerical Solutions (2)
- Special Functions (3)
- Sturm-Liouville Theory (2)

Second semester: (25 lectures)

- Nonhomogeneous Equations (2)
- Green's Functions - ODEs (2)
- Green's Functions - PDEs (2)
- Complex Variables (4)
- Fourier Transforms (3)
- Nonlinear PDEs (2)
- Other - numerical, conservation laws, symmetries (10)

An appendix is provided for reference, especially to basic calculus techniques, differential equations, and (maybe later) linear algebra.

Acknowledgments

MOST, IF NOT ALL, OF THE IDEAS AND EXAMPLES are not my own. These notes are a compendium of topics and examples that I have used in teaching not only differential equations, but also in teaching numerous courses in physics and applied mathematics. Some of the notions even extend back to when I first learned them in courses I had taken.

I would also like to express my gratitude to the many students who have found typos, or suggested sections needing more clarity in the core set of notes upon which this book was based. This applies to the set of notes used in my mathematical physics course, applied mathematics course, *An Introduction to Fourier and Complex Analysis with Application to the Spectral Analysis of Signals*, and ordinary differential equations course, *A Second Course in Ordinary Differential Equations: Dynamical Systems and Boundary Value Problems*, all of which have some significant overlap with this book.