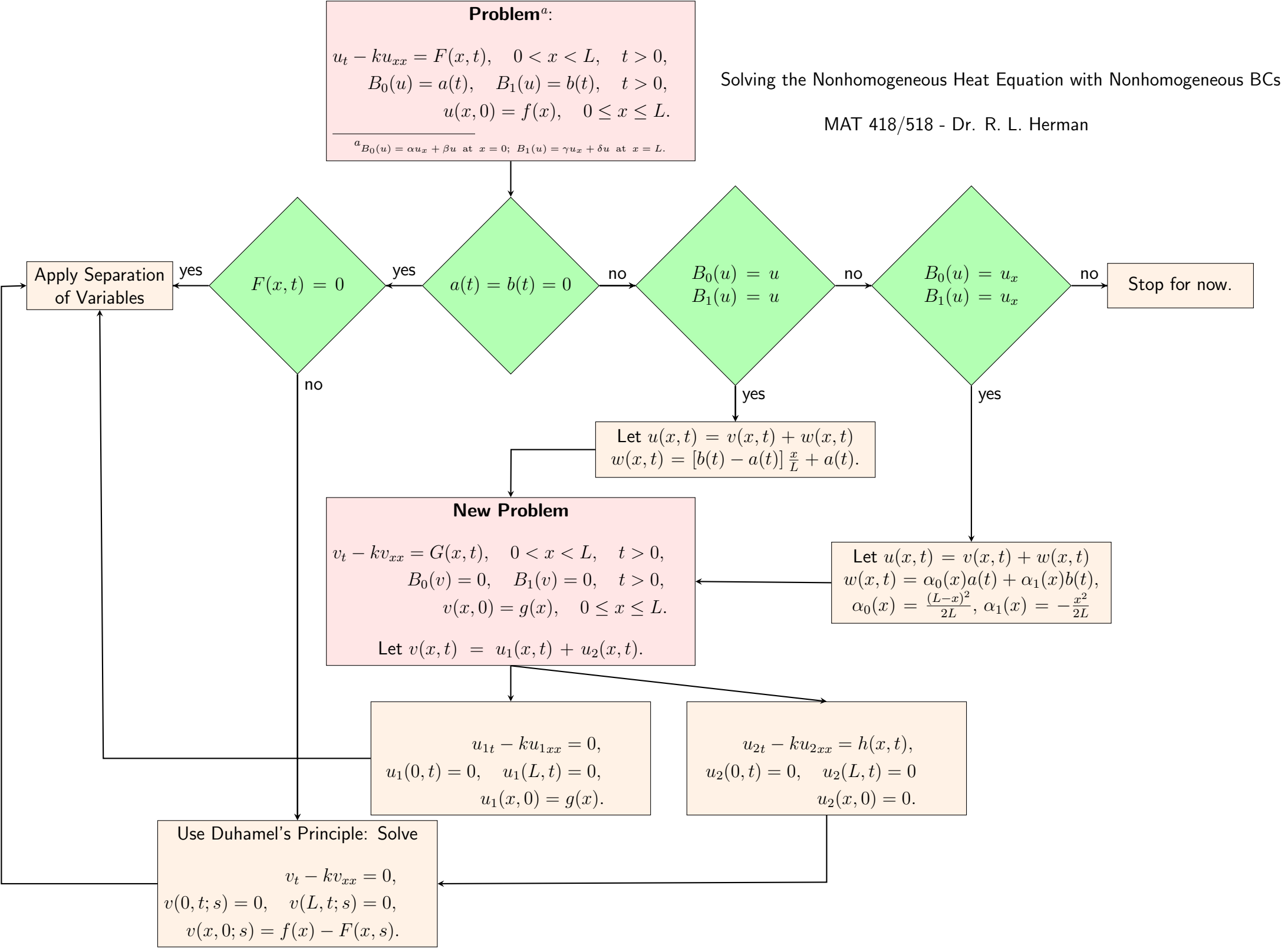
Problem[a]:
$u_t - ku_{xx} = F(x,t), \quad 0 < x < L, \quad t > 0,$
$B_0(u) = a(t), \quad B_1(u) = b(t), \quad t > 0,$
$u(x,0) = f(x), \quad 0 \le x \le L.$
[a] $B_0(u) = \alpha u_x + \beta u$ at $x = 0$; $B_1(u) = \gamma u_x + \delta u$ at $x = L$.
Solving the Nonhomogeneous Heat Equation with Nonhomogeneous BCs
MAT 418/518 - Dr. R. L. Herman
$F(x,t) = 0$
$a(t) = b(t) = 0$
$B_0(u) = u$
$B_1(u) = u$
$B_0(u) = u_x$
$B_1(u) = u_x$
yes
no
Apply Separation of Variables
Stop for now.
Let $u(x,t) = v(x,t) + w(x,t)$
$w(x,t) = [b(t) - a(t)]\frac{x}{L} + a(t).$
Let $u(x,t) = v(x,t) + w(x,t)$
$w(x,t) = \alpha_0(x)a(t) + \alpha_1(x)b(t),$
$\alpha_0(x) = \frac{(L-x)^2}{2L}, \ \alpha_1(x) = -\frac{x^2}{2L}$
New Problem
$v_t - kv_{xx} = G(x,t), \quad 0 < x < L, \quad t > 0,$
$B_0(v) = 0, \quad B_1(v) = 0, \quad t > 0,$
$v(x,0) = g(x), \quad 0 \le x \le L.$
Let $v(x,t) = u_1(x,t) + u_2(x,t).$
$u_{1t} - ku_{1xx} = 0,$
$u_1(0,t) = 0, \quad u_1(L,t) = 0,$
$u_1(x,0) = g(x).$
$u_{2t} - ku_{2xx} = h(x,t),$
$u_2(0,t) = 0, \quad u_2(L,t) = 0$
$u_2(x,0) = 0.$
Use Duhamel's Principle: Solve
$v_t - kv_{xx} = 0,$
$v(0,t;s) = 0, \quad v(L,t;s) = 0,$
$v(x,0;s) = f(x) - F(x,s).$