# Prologue

*"How can it be that mathematics, being after all a product of human thought independent of experience, is so admirably adapted to the objects of reality?." - Albert Einstein (1879-1955)*

## Introduction

THIS SET OF NOTES WAS COMPILED for use in a one semester course on mathematical methods for the solution of partial differential equations typically taken by majors in mathematics, the physical sciences, and engineering. Partial differential equations often arise in the study of problems in applied mathematics, mathematical physics, physical oceanography, meteorology, engineering, and biology, economics, and just about everything else. However, many of the key methods for studying such equations extend back to problems in physics and geometry. In this course we will investigate analytical, graphical, and approximate solutions of some standard partial differential equations. We will study the theory, methods of solution and applications of partial differential equations.

We will first introduce partial differential equations and a few models. A PDE, for short, is an equation involving the derivatives of some unknown multivariable function. It is a natural extenson of ordinary differential equations (ODEs), which are differential equations for an unknown function one one variable. We will begin by classifying some of these equations.

We begin the study of PDEs with the one dimensional heat and wave equations and the two-dimensional Laplace equation on a rectangle. As we progress through the course, we will introduce standard numerical methods since knowing how to numerically solve differential equations can be useful in research. We will also look into the standard solutions, including separation of variables, starting in one dimension and then proceeding to higher dimensions. This naturally leads to finding solutions as Fourier series and special functions, such as Legendre polynomials and Bessel functions. We will end with a short study of first order evolution equations.

The specific topics to be studied and approximate number of lectures will include

First Semester: (26 lectures)

- Introduction (1)

- Derivation of Generic Equations (1)
- Separation of Variables (Heat and Wave Equations) (2)
- 1D Wave Equation - d'Alembert Solution (2)
- Classification of Second Order Equations (1)
- Nonhomogeneous Heat Equation (1)
- Separation of Variables (2D Laplace Equation) (2)
- Fourier Series (4)
- Finite Difference Method (2)
- Sturm-Liouville Theory (3)
- Special Functions (3)
- Equations in 2D - Laplace's Equation, Vibrating Membranes (3)
- 3D Problems and Spherical Harmonics (2)
- First Order PDEs (2)
- Conservation Laws and Shocks (1)

## *Acknowledgments*

MOST, IF NOT ALL, OF THE IDEAS AND EXAMPLES are not my own. These notes are a compendium of topics and examples that I have used in teaching not only differential equations, but also in teaching numerous courses in physics and applied mathematics. Some of the notions even extend back to when I first learned them in courses I had taken.

I would also like to express my gratitude to the many students who have found typos, or suggested sections needing more clarity in the core set of notes upon which this book was based. This applies to the set of notes used in my mathematical physics course, applied mathematics course, and previous differential equations courses.