

# Index

- adjoint operator, 102
- area element, 208
- associated Laguerre polynomials, 201
- associated Legendre functions, 131, 185
- average of a function, 52, 174
  
- backward difference, 84
- Bernoulli, Daniel, 3, 135
- Bessel functions, 135, 160
  - first kind, 136
  - Fourier-Bessel series, 138
  - generating function, 138
  - identities, 137
  - orthogonality, 138
  - recursion formula, 137
  - second kind, 137
  - spherical, 194
- Bessel's inequality, 142
- Bessel, Friedrich Wilhelm, 135
- Bohr radius, 201
- boundary conditions, 3, 97
  - nonhomogeneous, 33
  - time dependent, 37
  - time independent, 33
- boundary value problem, 4, 215
- breaking time, 228
- Burgers' equation, inviscid, 226
  
- cakes
  - baking, 175
  - cylindrical, 178
  - rectangular, 175
- central difference, 84
- characteristics
  - fan, 233
- Charpit equations, 241
- circular membrane, 156
- Clairaut's Theorem, 216
- Clairaut, Alexis Claude, 216
- classical orthogonal polynomials, 119, 122
  
- classification of PDEs, 27
- complete basis, 142
- conics, 32
- conservation law
  - differential form, 225
  - integral form, 225
- conservation laws., 224
- convergence in the mean, 142
- cooking
  - cakes, 175
  - turkey, 193
- coordinates
  - curvilinear, 203
  - cylindrical, 207
  - spherical, 204
- cosine series, 64
- curvilinear coordinates, 203
- cylindrical coordinates, 207
  
- d'Alembert's formula, 16
- d'Alembert, Jean le Rond, 3
- d'Alembert, Jean-Baptiste, 16
- differential equations
  - series, 136
  - series solutions, 200
- Dirichlet boundary conditions, 97
- Dirichlet kernel, 77
- Dirichlet, Gustav, 97
- double factorial, 127
- Duhamel's Principle, 39, 45
- Duhamel, Jean-Marie Constant, 39
  
- easy as  $\pi$ , 161
- eigenfunction expansion, 107
- eigenfunctions
  - complete, 99
  - orthogonal, 99
- eigenvalues
  - real, 99
- Einstein, Albert, vii
- electric field, 41

- electric potential, 41
- electrostatics, 41
- elliptic equation, 29
- equilibrium solution, 41
- equilibrium temperature, 168
- Euler, Leonhard, 3, 135
- even functions, 58
  
- fan characteristics, 233
- finite difference method, 83
- fluid flow, 41
- forward difference, 84
- Fourier analysis, 2, 50
- Fourier coefficients, 51
  - generalized, 108, 118
- Fourier series, 50
  - generalized, 118
  - Maple code, 67
  - representation on  $[0, 2\pi]$ , 51
- Fourier, Joseph, 3, 135, 215
- Fourier-Bessel series, 135
- Fourier-Legendre series, 122, 131
- Fredholm alternative, 110
- function spaces, 116
  
- Gödel, Kurt, 1
- Gamma function, 133
- Gegenbauer polynomials, 122
- Gibbs phenomenon, 67, 74
- Gibbs, Josiah Willard, 74
- Gram-Schmidt Orthogonalization, 119
- gravitational potential, 126
- Green's identity, 104
  
- harmonic functions, 41, 182
- harmonics, 3
  - meridional, 190
  - sectoral, 190
  - spherical, 190
  - surface, 190
  - tesseral, 190
  - zonal, 190
- heat equation, 1
  - 1D, 10, 69
  - derivation, 9
  - nonhomogeneous, 33
- Heaviside function, 132
- Heaviside, Oliver, 132
- Helmholtz equation, 150
- Helmholtz, Hermann Ludwig Ferdinand
  - von, 150
- Hermite polynomials, 122, 144
- hydrogen atom, 198
- hyperbolic equation, 29
  
- incompressible, 41
- infinite dimensional, 116
- infinitesimal area element, 208
- infinitesimal volume elements, 207
- initial value problem, 215
- inner product, 116
- inner product spaces, 117
- integration by parts, 58
- inviscid Burgers' equation, 226
- irrotational, 41
  
- Jacobi polynomials, 122
- Jacobian determinant, 207
  
- Kepler, Johannes, 3, 135
- kettle drum, 156
- Kronecker delta, 117
  
- Lagrange's identity, 104
- Lagrange, Joseph-Louis, 135
- Lagrange-Charpit method, 241
- Laguerre polynomials, 122, 201
- Laguerre, Edmond, 201
- Laplace's equation
  - Polar coordinates, 172
  - Rectangular coordinates, 41, 166
  - rectangular plates, 168
  - spherical symmetry, 182
  - two dimensions, 40, 165
- Laplace, Pierre-Simon, 182
- Laplacian, 1
  - cylindrical, 211
  - polar coordinates, 157
  - spherical, 211
- least squares
  - approximation, 139
- Legendre polynomials, 122, 185
  - generating function, 126
  - leading coefficient, 125
  - normalization, 130
  - recurrence relation, 124
  - recursion formula, 124
  - Rodrigues formula, 123
- Legendre, Adrien-Marie, 122, 133
- line element, 206
  
- mean square deviation, 140
- mean value theorem, 174
- membrane
  - annular, 164
  - circular, 156
  - rectangular, 151
- Method of Characteristics, 219
- metric coefficients, 206

- Monge, Gaspard, 241
- Neumann boundary conditions, 97
- Neumann function, 137, 160
- Neumann, Carl, 97
- Newton's Law of Cooling, 213
- nodal curves, 153
- nodal lines, 153
- norm of functions, 119
- normalization, 53, 118
- numerical stability, 91
- odd functions, 58
- ordinary differential equation, 215
- orthogonal functions, 53
- orthogonal grid, 204
- orthogonality
- eigenfunctions, 99
- orthonormal, 53
- Panofsky, Pief, 192
- parabolic equation, 29
- Parseval's equality, 142
- Parseval's identity, 79
- partial differential equation, 1
- period, 49
- periodic boundary conditions, 97, 159
- periodic extension, 51, 60
- periodic function, 49
- phase, 50
- phase shift, 50
- Poisson Integral Formula, 174
- Poisson kernel, 174
- Poisson, Siméon, 135
- polar coordinates
- grid, 205
- product solutions, 13
- quadratic forms, 32
- Rankine-Hugoniot jump condition, 231
- Rayleigh quotient, 106
- Rayleigh, Lord, 100
- real eigenvalues, 99
- rectangular membrane, 151
- rescaling, 193
- Riemann-Lebesgue Lemma, 142
- Russell, Bertrand, 47
- scalar product, 53
- scale factors, 205
- Schrödinger equation, 196
- self-adjoint, 103
- self-similarity transformation, 193
- separation of variables, 10
- series
- Fourier series, 50
- side conditions, 221
- sine series, 64
- double, 155
- source density, 225
- space of square integrable functions, 142
- spectral analysis, 2
- spherical Bessel functions, 194
- spherical harmonics, 183, 188
- spherical symmetry, 182
- spherical turkey, 192
- stability
- numerical, 91
- steady state solution, 34
- stencil, 85
- Stone-Weierstraß Theorem, 119
- Strutt, John William, 100
- Sturm-Liouville operator, 96
- Sturm-Liouville problem, 95, 138, 145, 146
- Taylor, Brook, 3
- Tchebychef polynomials, 122
- Thanksgiving, 192
- thermal equilibrium, 41
- tones, 48
- traffic flow, 226
- traveling wave, 221
- trigonometric identities, 52
- truncation error, 90
- turkey, 192
- vector spaces
- function spaces, 116
  - inner product space, 117
- vectors
- projection, 120
- vibrating sphere, 190
- vibrating string, 2
- Villocourt, Paul Charpit, 241
- volume elements, 207
- von Neumann, John, 83
- Wallis, John, 3
- Watson, G.N., 135
- wave equation, 2, 3
- 1D, 14, 72
  - circular, 156
  - derivation, 7
  - rectangular, 151
- weak solutions, 230
- Wilbraham, Henry, 74
- Wilkinson, Rebecca, 175