MAT 518 Graduate Midterm **Name ________________________**

Instructions:

- Place your name on all of the pages.
- Do all of your work in this booklet. Do not tear off any sheets.
- Show all of your steps in the problems for full credit.
- Be clear and neat in your work. Any illegible work, or scribbling in the margins, will not be graded.
- Put a box around your answers when appropriate.
- If you need more space, you may use the back of a page and write *On back of page #* in the problem space or the attached blank sheet. **No other scratch paper is allowed.**

Try to answer as many problems as possible. Provide as much information as possible. Show sufficient work or rationale for full credit. Remember that some problems may require less work than brute force methods.

If you are stuck, or running out of time, indicate as completely as possible, the methods and steps you would take to tackle the problem. Also, indicate any relevant information that you would use. Do not spend too much time on one problem. **Pace yourself.**

Pay attention to the point distribution. Not all problems have the same weight.

Page	Pts	Score
1	16	
2	9	
3	14	
4	11	
Total	50	

Bonus: Solve $x^2y''+xy'-4y=0,\ y(1)=1, y(2)=0.$

MAT 518 Graduate Midterm **Name ________________________**

1. (4 pts) Consider the vibrating string solution, $u(x,t) = \sum_{n=1}^{\infty} c_n \sin 3nx \cos 9nt,$

   a. Show this is the sum of left and right traveling waves.

   b. What is the wave speed? ___________

2. (4 pts) Find the eigenvalues, λ, and eigenfunctions, $y(x)$, for the boundary value problem: $y'' + y = \lambda y,\ y(0) = 0, y'(\pi) = 0.$

3. (3 pts) Given the initial conditions $u(x,0) = \sin(\pi x) - 0.5\sin(5\pi x),$ $u_t(x,0) = 0,\ x \in [0,1]$ for a vibrating string with two fixed ends and wave speed $c = 3,$, write down the solution $u(x,t)$.

4. (5 pts) Classify the equations over regions of the *xy*-plane.

   a. $u_{xx} + 2u_{xy} + u_{yy} + u_x = 0.$ _________________

   b. $u_{xx} + 2u_{xy} + \frac{1}{2}u_{yy} = 0.$ _________________

   c. $2u_{xx} + 6u_{xy} + 5u_{yy} + u_x = 0.$ _________________

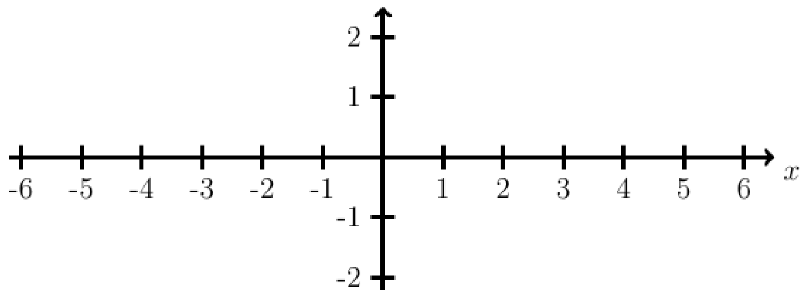
   d. $xu_{xx} - yu_{yy} = 0.$ _________________

MAT 518 Graduate Midterm **Name ________________________**

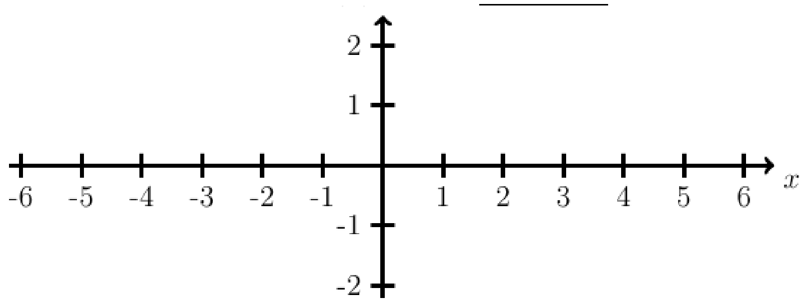
5. (9 pts) Let $f(x) = 2 - \dfrac{x^2}{2}, 0 \le x \le 2.$

   a. Sketch <u>several</u> periods of the following and <u>state the period</u>:

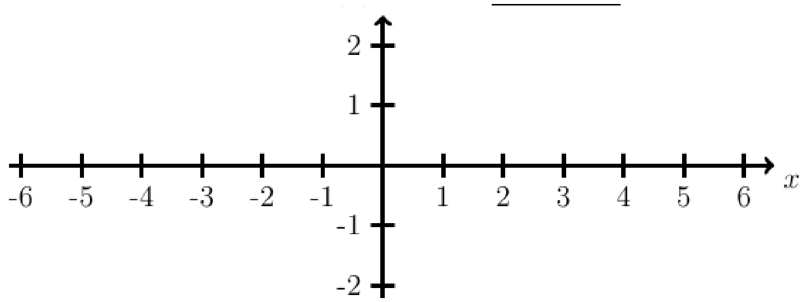
      i. The periodic extension. Period = ________



      ii. The odd extension. Period = ________



      iii. The even extension. Period = ________



   b. Find the Fourier Sine series, $f(x) \sim \sum_{n=1}^{\infty} b_n \sin \dfrac{n\pi x}{2}$, for this function.

MAT 518 Graduate Midterm **Name ________________________**

6. (5 pts) Find the Fourier series representation of $f(x) = \begin{cases} x, & 0 < x < 1, \\ 0, & 1 < x < 2. \end{cases}$

7. (5 pts) Consider Laplace's equation, $u_{xx} + u_{yy} = 0,$ on the unit square satisfying the boundary conditions: $u_x(0, y) = 0,\ \ u(x,0) = 0 = u(x,1).$
   a. Carry out a separation of variables and give the differential equations for the product functions, $X(x)$ and $Y(y)$.

   b. Write down the product solutions, $u_n(x, y),$ satisfying the boundary conditions without doing a lot of work.

8. (4 pts) Consider the finite difference approximation $\dfrac{u(x) - 2u(x-\Delta x) + u(x-2\Delta x)}{\Delta x^2}$.

   Use Taylor series to determine what this approximates.

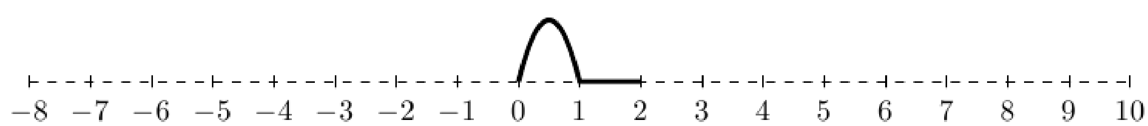
9. (5 pts) Consider the problem

$$u_t = k u_{xx},\ 0 \le x \le 1,\ t \ge 0.$$
$$u(0,t) = 1,\ u(1,t) = 5.$$
$$u(x,0) = 4x + 3\sin 5\pi x + 1.$$

   a. Find the steady state solution.

   b. Use the steady state solution to write down the solution to the full problem. [If you did this correctly, you do not need a lot of work to write down the answer.]

10. (3 pts) A string of length two has one fixed end, $u(0,t) = 0,$ and one free end, $u_x(2,t) = 0.$ The initial profile is shown below. Fill the plot below to represent the proper extension for use in d'Alembert's solution of the wave equation.



11. (3 pts) Solve: $u_t - u_{xx} = t\sin 3x \quad 0 < x < \pi,\ t > 0,$ satisfying the conditions $u(0,t) = 0,\ u(\pi,t) = 0,\ t > 0,$ and $u(x,0) = 0,\ 0 < x < \pi.$

Extra