MAT 418/518 Review

A. General Methods

1) Separation of Variables: Know how to find/ write standard solutions to various boundary value problems (sometimes quickly);
2) Fourier Series: Know basic formulae for trigonometric, sine, and cosine series. Be able to identify and sketch appropriate extensions for various boundary conditions.
3) General Fourier Series $f(x)=\sum_{n=1}^{\infty} c_n\phi_n(x),\ c_n=\dfrac{(f,\phi_n)}{\|\phi_n\|^2}$
4) Gram-Schmidt Orthogonalization

B. Equations in 1D, 2D, 3D

1) Wave Equation
   i. D'Alembert's Solution
   $$u(x,t)=\frac{1}{2}[f(x-at)+f(x+at)]+\frac{1}{2a}\int_{x-at}^{x+at} g(r)\,dr$$
   ii. Vibrational Modes of Rectangular and Circular Membranes
2) Heat Equation
3) Helmholtz Equation
4) Laplace's Equation
   i. Dirichlet Problem on Disk $u(r,\theta)=\dfrac{a_0}{2}+\sum_{n=1}^{\infty}\left(\dfrac{r}{r_o}\right)^n(a_n\cos n\theta+b_n\sin n\theta),\ r<r_o.$

C. Boundary Conditions

1) Dirichlet - fixed
2) Neumann – free or insulated
3) Periodic Boundary Conditions

D. Geometries in 1D-3D

1) Know types of solutions for different geometries and when they appear such as trigonometric, hyperbolic,
   i. Bessel Functions
   ii. Legendre Functions
   iii. Spherical Harmonics, $Y_{\ell m}(\theta,\varphi)=P_\ell^m(\cos\theta)e^{\pm im\varphi}$.
2) Rectangular $\nabla^2 u=\dfrac{\partial^2 u}{\partial x^2}+\dfrac{\partial^2 u}{\partial y^2}+\dfrac{\partial^2 u}{\partial z^2}$
3) Polar $\nabla^2 u=\dfrac{1}{r}\dfrac{\partial}{\partial r}\left(r\dfrac{\partial u}{\partial r}\right)+\dfrac{1}{r^2}\dfrac{\partial^2 u}{\partial\theta^2}$
4) Cylindrical $\nabla^2 u=\dfrac{1}{r}\dfrac{\partial}{\partial r}\left(r\dfrac{\partial u}{\partial r}\right)+\dfrac{1}{r^2}\dfrac{\partial^2 u}{\partial\theta^2}+\dfrac{\partial^2 u}{\partial z^2}$
5) Spherical $\nabla^2 u=\dfrac{1}{\rho^2}\dfrac{\partial}{\partial\rho}\left(\rho^2\dfrac{\partial u}{\partial\rho}\right)+\dfrac{1}{\rho^2\sin\phi}\dfrac{\partial}{\partial\phi}\left(\sin\phi\dfrac{\partial u}{\partial\phi}\right)+\dfrac{1}{\rho^2\sin^2\phi}\dfrac{\partial^2 u}{\partial\theta^2}$

E. Sturm-Liouville Problems

1) Put ODEs in Sturm-Liouville Form $Ly=\dfrac{d}{dx}\left(p\dfrac{dy}{dx}\right)+qy=-\lambda\sigma y.$
2) Lagrange Identity and Green's Formula $uLv-vLu=\dfrac{d}{dx}[puv'-pu'v]_a^b$
3) Properties of Sturm-Liouville Eigenvalue Problems
4) Proof that eigenvalues are real and eigenfunctions are orthogonal
5) Using Rayleigh's Formula

F. First Order PDEs

1) Linear and Quasilinear PDEs, Method of Characteristics
2) Shock and Rarefaction Waves

G. Other

1) Gamma function and factorials
2) Simple ODEs
3) Know BCs: Fixed, Dirichlet, Insulating, Neumann, Robin, Periodic.