# MAT 463 Practice EX III

1. Find the eigenfunctions and eigenvalues of the boundary value problem:
   $x''(t) + k^2 x(t) = 0, \quad x(-\pi) = 0, \quad x'(\pi) = 0.$

2. Show that the set $\{\sin 3nx\}, n = 1,2,3,\ldots$ is orthogonal over the interval $[-\pi, \pi]$.

3. Find the Fourier series expansion of the square wave given by $f(x) = (-1)^n k$, $n < x < n+1$, where $k$ is a constant. Use your result to obtain a series expansion for $\pi$.

4. Find the eigenfunctions and eigenvalues of the BVP
   $x^2 y'' + 3xy' + (\lambda + 1)y = 0, \; y(1) = y(e) = 0.$

5. Consider the heat equation $\dfrac{\partial u}{\partial t} = \dfrac{\partial^2 u}{\partial x^2}, \quad 0 < x < 1, \quad t > 0.$ This describes the temperature $u(x,t)$ of a one-dimensional rod of length one. Given that the temperature is fixed at both ends, $u(0,t) = 0 = u(1,t), t > 0$, and the initial temperature is given by $u(x,0) = \sin 3\pi x$, determine the temperature at later times. Describe what happens for long times.

6. Consider the function $f(x) = (x^2 - 1)^2$ on $[-1,1]$.
   a. Find the Fourier Coefficients.

   b. Use the result of part (a) to find the sum of the infinite series $\sum_{n=1}^{\infty} \dfrac{1}{n^4}$.

7. Determine the Fourier coefficients for $f(x) = 3\sin 2x - \cos^2 x, \; x \in [0, 2\pi]$.

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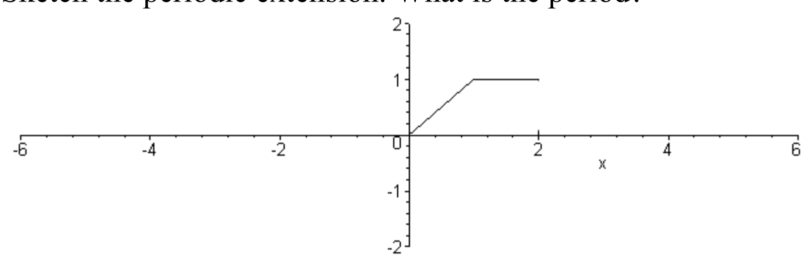
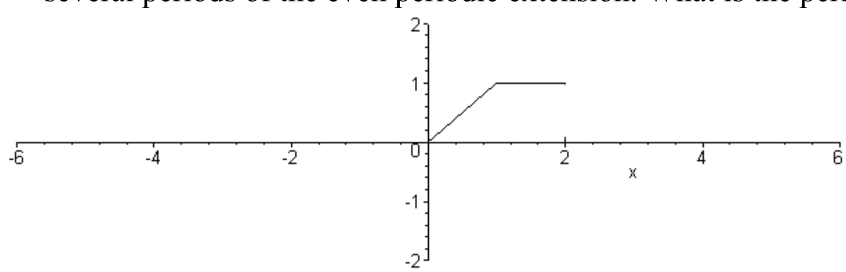
8. Let $f(x) = \begin{cases} x, & 0 < x < 1 \\ 0, & 1 < x < 2 \end{cases}$.
    a. Sketch several periods of the following:
        i. The periodic extension of period 2.
        ii. The even extension.
        iii. The odd extension.
    b. Find the Fourier cosine series for this function.

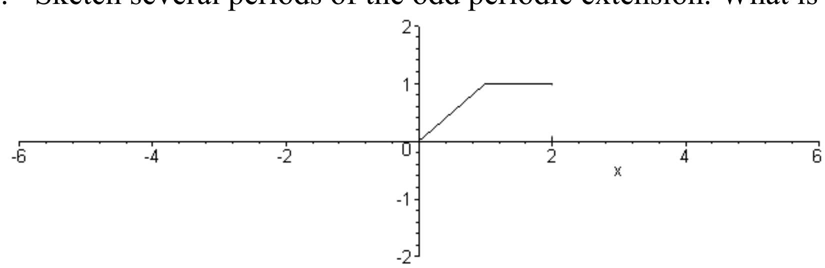
9. Consider the function graphed in each part below.
    a. Sketch the periodic extension. What is the period?




    b. Sketch several periods of the even periodic extension. What is the period?




    c. Sketch several periods of the odd periodic extension. What is the period?




10. Given $f(x) = |x|$,
    a. Find the Fourier trigonometric series of $f(x)$ over $-\pi < x < \pi$.
    b. Find the Fourier sine series of $f(x)$ over $0 < x < \pi$.