MAT 463/563 Ch 4 and 5 Review

I. Preliminary Material
    a. Constant Coefficient Equations
    b. Cauchy-Euler Equations
II. Boundary Value Problems
    a. Direct Solution
    b. Separation of Variables – Heat Equation
    c. Solution of Eigenvalue Equations with Homogeneous BCs
III. Vector Spaces and Function Spaces
    a. Bases/Eigenfunctions
    b. Scalar Product $<f,g> = \int_a^b f(x)g(x)\sigma(x)\,dx$
    c. Orthogonal, Orthonormal, Normalization
    d. Orthogonality of $\{1, \cos\frac{\pi x}{L}, \cos\frac{2\pi x}{L}, \ldots, \sin\frac{\pi x}{L}, \sin\frac{2\pi x}{L}, \ldots\}$ on $[-L, L]$, etc
    e. Determination of Expansion Coefficients in basis expansions
IV. Fourier Series
    a. Trigonometric Fourier Series Expansion on $[-\pi, \pi]$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx + b_n \sin nx$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos nx\,dx, \quad b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin nx\,dx$$

   b. Trigonometric Fourier Series Expansion on $[0, 2\pi]$: $a_n = \frac{1}{\pi}\int_0^{2\pi} f(x)\cos nx\,dx$, etc.

   c. Trigonometric Fourier Series Expansion on $[-L, L]$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\frac{n\pi x}{L} + b_n \sin\frac{n\pi x}{L}$$

$$a_n = \frac{1}{L}\int_{-L}^{L} f(x)\cos\frac{n\pi x}{L}\,dx, \quad b_n = \frac{1}{L}\int_{-L}^{L} f(x)\sin\frac{n\pi x}{L}\,dx$$

   d. Half Range Expansions on $[0, L]$.

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\frac{2n\pi x}{L} + b_n \sin\frac{2n\pi x}{L}$$

$$a_n = \frac{2}{L}\int_0^L f(x)\cos\frac{2n\pi x}{L}\,dx, \quad b_n = \frac{2}{L}\int_0^L f(x)\sin\frac{2n\pi x}{L}\,dx$$

   e. Fourier Sine and Cosine Series

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\frac{n\pi x}{L}, \quad a_n = \frac{2}{L}\int_0^L f(x)\cos\frac{n\pi x}{L}\,dx,$$

   f. Even Periodic Extension, Odd Periodic Extension, Periodic Extension
V. Nonautonomous systems, Duffing Equation, Surface of Section, Strange Attractors.