0.1 Problem Set 3

1. Consider the following systems. For each system determine the coefficient matrix. When possible, solve the eigenvalue problem for each matrix and use the eigenvalues and eigenfunctions to provide solutions to the given systems. Finally, in the cases which you already investigated in Problem Set 2, make comparisons with your previous answers.

(a)

$$x' = 3x - y$$
$$y' = 2x - 2y.$$

(b)

$$x' = -y$$
$$y' = -5x.$$

(c)

$$x' = x - y$$
$$y' = y.$$

(d)

$$x' = 2x + 3y$$
$$y' = -3x + 2y.$$

(e)

$$x' = -4x - y$$
$$y' = x - 2y.$$

(f)

$$x' = x - y$$
$$y' = x + y.$$

2. Consider the system $\mathbf{x}' = A\mathbf{x}$ where

$$A = \begin{pmatrix} 1 & -1 & 4 \\ 3 & 2 & -1 \\ 2 & 1 & -1 \end{pmatrix}. \tag{1}$$

(a) Find the fundamental solution matrix.

(b) Find the principal solution matrix.