MAT 463/563 Exam III   Name _____

Instructions:

- Place your name on all of the pages.
- Do all of your work in this booklet. Do not tear off any sheets.
- Show all of your steps in the problems for full credit.
- Be clear and neat in your work. Any illegible work, or scribbling in the margins, will not be graded.
- Put a box around your answers when appropriate..
- If you need more space, you may use the back of a page and write *On back of page #* in the problem space or the attached blank sheet. **No other scratch paper is allowed.**

Try to answer as many problems as possible. Provide as much information as possible. Show sufficient work or rationale for full credit. Remember that some problems may require less work than brute force methods.

If you are stuck, or running out of time, indicate as completely as possible, the methods and steps you would take to tackle the problem. Also, indicate any relevant information that you would use. Do not spend too much time on one problem. **Pace yourself**

Pay attention to the point distribution. Not all problems have the same weight.

Page	Pts	Score
1	18	
2	14	
3	18	
Total	50	

Bonus: Use the result of Problem 5 to find a series expansion for π^3

1. (3 Pts) Show that the set of functions $\{\cos n\pi x\}, \ n = 0,1,2,\ldots$ is orthogonal on $0 \le x \le 2.$

2. (5 Pts) Find the eigenvalues, λ, and eigenfunctions, $y(x)$, for the eigenvalue problem
   $y'' + \lambda y = 0, \quad y'(0) = 0, \ y(\pi) = 0.$

3. (5 Pts) Find the eigenvalues, λ, and eigenfunctions, $y(x)$, for the eigenvalue problem
   $(xy')' + \dfrac{\lambda}{x} y = 0, \quad y(1) = 0, \ y(e) = 0.$

4. (5 Pts) Solve the boundary value problem for $y(x)$: $y'' + 4y' = 0$, $y'(0) = 1$, $y(1) = 0.$

5. (5 Pts) Consider the function $f(x) = x^2 - 2$ on $[0,2]$. Find the Fourier sine series for $f(x)$.

6. (3 Pts) Without integrating, find the Fourier coefficients of $f(x) = 2 + 4\sin^2 3x - \cos 5x$ on the interval $x \in [-\pi, \pi]$.

7. (6 Pts) Consider the set $\{1, x, x^2, \ldots\}$ on the interval $[-1,1]$.
   a. Show that $g(x) = 1$, and $f(x) = x^2$ are not orthogonal.

   b. Find the norm of $f(x) = x^2$

8. (18 Pts) Let $f(x) = \begin{cases} x, & 0 < x < \pi \\ 2\pi - x, & \pi < x < 2\pi \end{cases}$.

    a. Sketch several periods of the following:

        i. The periodic extension.

        ii. The even periodic extension.

        iii. The odd periodic extension.

    b. Find the Fourier series for this function.

Extra Space