MAT 463/563 Midterm **Name** _____

Instructions:

- Place your name on all of the pages.
- Do all of your work in this booklet. Do not tear off any sheets.
- Show all of your steps in the problems for full credit.
- Be clear and neat in your work. Any illegible work, or scribbling in the margins, will not be graded.
- Put a box around your answers when appropriate..
- If you need more space, you may use the back of a page and write *On back of page #* in the problem space or the attached blank sheet. **No other scratch paper is allowed.**

Try to answer as many problems as possible. Provide as much information as possible. Show sufficient work or rationale for full credit. Remember that some problems may require less work than brute force methods.

If you are stuck, or running out of time, indicate as completely as possible, the methods and steps you would take to tackle the problem. Also, indicate any relevant information that you would use. Do not spend too much time on one problem. **Pace yourself – do not spend more than 15 minutes per page on your first pass.**

Pay attention to the point distribution. Not all problems have the same weight.

Page	Pts	Score
1	**15**	
2	**13**	
3	**25**	
4	**12**	
5	**Bonus**	
Total	**65**	

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1. (10 pts) Solve the following equations:
   a. $xy' = y^2,\ y(1) = 1.$

   b. $xy' - y = x^3 e^x.$

   c. $x^2 y'' + 3xy' + y = 3x^6.$

2. (5 pts) Two large tanks, each holding 10 liters of a saltwater solution are interconnected by pipes. Fresh water flows into tank A at a rate of 5 liters per minute, and fluid is drained out of tank B at the same rate. The tanks have two pipes connecting them which allow for exchange of fluid at the following rates: from A to B at 7 liters per minute and from B to A at 3 liters per minute. The solution in tank A contains 25 grams of salt, and the solution in tank B contains 50 grams of salt. Model the system of equations describing changes in the salt content in the tanks over time. Do not solve this system.

3. (13 pts) In the following problems locate and classify the equilibrium (fixed) points of the system.

a.
$$x' = 4x - y$$
$$y' = 6x + 3y$$

b.
$$x' = x - 3y$$
$$y' = x - y$$

c.
$$x' = y(1 - x^2)$$
$$y' = 1 - y^2$$

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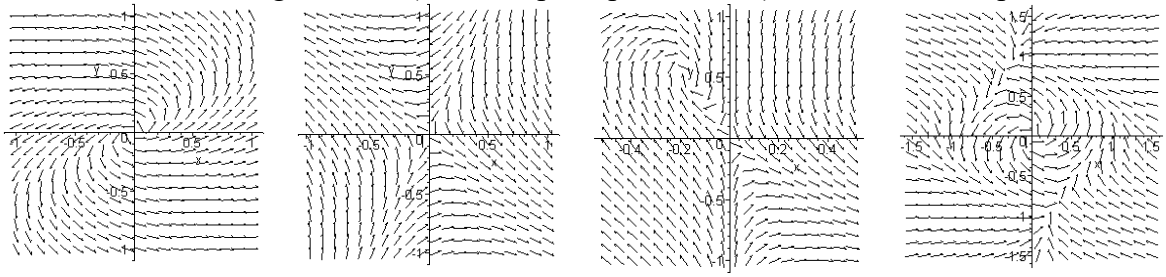
4. **Not Now** (10 pts) A simple population model is one of harvesting in a fish farm. Let's assume that such a model, which includes harvesting, is $y' = (6 - y)y - h.$ Here we take *y(t)* to be thousands of fish in week *t. h* is the number of fish in thousands that are harvested per week.
    a. Let $h = 5.$ Describe the fish farm business following this model in which the owners start with 750 fish. What happens to a business starting with 2500 fish? Support your answers by plotting a sufficient number of solutions in the *ty*-plane.

    b. Construct a bifurcation diagram *y* vs. *h* for the general model and identify the bifurcation point.

5. (15 pts) In the following figures describe the types of behavior you see and sketch typical orbits supporting your observations. For each fixed point you find, describe the form of the eigenvalues (real, complex, positive, etc) of the linearized problem.



A B C D

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6. (12 pts) Consider the system $\begin{aligned} x' &= -2x + y \\ y' &= 4x + y \end{aligned}$.

   a. Find the eigenvalues of the coefficient matrix.

   b. Find the eigenvectors of the coefficient matrix.

   c. Construct the Fundamental matrix.

   d. Write the general solution.

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Additional Problems not on this exam

7. In the following problems locate and classify the equilibrium (fixed) points of the system.
   a. $x' = -x + 2x^3$, $y' = 2y + y^3$
   b. $x' = -x - x\sin x$, $y' = \sin y$
   c. $x' = y(1 - x^2)$, $y' = 1 - y^2$
8. Construct bifurcation diagrams for the parameter in the equation.
   a. $x' = rx + 4x^3$
   b. $x' = x - \mu x(1 - x)$
9. Let $x' = x - y - x(x^2 + y^2)$, $y' = x + y - y(x^2 + y^2)$. Convert to polar form and describe the type of orbits expected.