 Spring 2025
Revised: March 26, 2025

# Gibbs Phenomenon - History

**Abstract**

This is a draft of a history of the Gibbs Phenomenon. There have been several historical reviews, often buried in articles or books. However, this might be useful to have links between authors as well at to online resources, to sort out some aspects of the history leading to the more general theory.

## Introduction

Here is an early listing for the history of the Gibbs phenomena:

- It was coined by Bôcher in his monograph (Bôcher 1906) as he noted in a later paper, (Bôcher 1914), online.
- Bôcher mentions a monograph by H. Burkhardt, *Entwicklungen nach oscillirenden Functionen* as having a history of the vibrating string controversy. (Burkhardt 1908).
- Henry Wilbraham (1848) discovered the phenomenon found here, pg 198, but was not noticed until mentioned in Heinrich Burkhardt's review (above).
  - Wilbraham also cited (Newman 1848), found here, pg 108, who said "what is true within the limits is true at the limits" is erroneous. Newman cited (De Morgan 1836), *Differential Calculus,* near page 607.
  - Francis W. Newman (1805-1897) said that Fourier indicated that the locus of points

    $$y = \cos x - \frac{1}{3}\cos 3x + \frac{1}{5}\cos 5x - \dots$$

    is a connected series of straight lines alternating parallel and perpendicular to the axes. Newman points to de Morgan's discussion which is supposedly modeling Fourier.
  - Jean-Baptiste Joseph Fourier (1768-1830) wrote a treatise in 1822 on heat flow. It was translated into English 56 years later. [Note: Thomson (1939, 1841), a.k.a. Lord Kelvin, first wrote on Fourier's work when he was a teenager. ]
    - Fourier's 1807 Memoir (Fourier 1808)
    - (Fourier 1822) Fourier's Théorie analytique de la chaleur, online
      (Fourier 1878)French1878 English Translation
      In the English translation, we find on page 143 the result of determining the coefficients $a, b, c, d$, &c. in the equation [Section II, subsection 171, and page 167 of Fourier (1822).]

      $$1 = a\cos y + b\cos 3y + c\cos 5y + d\cos 7y - \&c.$$

      The result Fourier found algebraically was

      $$\frac{\pi}{4} = \cos y - \frac{1}{3}\cos 3y + \frac{1}{5}\cos 5y - \frac{1}{7}\cos 7y + \frac{1}{9}\cos 9y - \&c.$$

Then, Alexander Freeman (1838–1897), the translator, referred to the Gregory (1838) article in a footnote. [See it here.] D. F. Gregory (1813-1844), who was first editor of the Cambridge Mathematical Journal, determined Fourier coefficients by integration. Note that Fourier algebraically solved for the first few coefficients.

  - de Morgan in the 1836 edition of *The differential and integral calculus* derives on pages 242-3

$$\frac{1}{2}\theta + m\pi = \sin\theta + \frac{\sin 2\theta}{2} + \frac{\sin 3\theta}{3} + \ldots.$$

  He then says,

  > The meaning of the undetermined quantity $m$ may easily be shown. The second side of the equation is periodic, giving the same values for $\theta, \theta + 2\pi, \theta + 4\pi, \&c.$ It also vanishes with $\theta$, and becomes $1 - \frac{1}{3} + \frac{1}{5} \ldots$, or $\frac{1}{4}\pi$, when $\theta = \frac{1}{2}\pi$, and changes sign with $\theta$; and it becomes 0 again when $\theta = \pi$. This requires that $m$ should $= 0$, where $\theta$ lies between $-\pi$ and $+\pi$; but that in all other cases $m$ should have such a value as will make $\theta + m\pi$ lie between $-\pi$ and $+\pi$.

  - Francis W. Newman wrote on Sine integral in (Gregory et al. 1847), page 75.

- A series of letters were inspired by A. Michelson's Oct 6 letter on Fourier Series (Michelson 1898).
  - Oct 13, 1898 A. E. H. Love (1898a) responded first.
  - Dec 29, Gibbs 1898 wrote a letter (online) after Michelson's reply to Love. Included afterwards was Love's answer which continued to another page. The full text of both can also be seen (Love 1898b) here.
  - Gibbs (1899) noted a correction on April 27, 1899. Here he makes the distinction between "the limit of the graphs, and ... the graph of the limit of the sum."
  - May 18, 1899 Michelson (1899) even posted a comment from Poincaré.
  - June 1, 1899 Love replied again (Love 1899).
  - Note that *Nature*, April 1899, was founded and edited by Norman Lockyer 1869-1920.
- (Hewitt and Hewitt 1979) Paper on The Gibbs-Wilbraham Phenomenon, online. Jerri 1998 quoted this in his book on Gibbs Phenomenon. Gottlieb and Shu (1997) repeats the history and gives complete references, online.
- (Byerly 1893) Early text on Fourier Series, online.
- Michelson and Stratton (1898) Harmonic Analyzer, online

H. S. Carslaw wrote a couple of papers to correct the history, Carslaw (1925a, 1925b). Carslaw (1917) also wrote earlier - Carslaw's 1917 paper online and Carslaw (1921) provided a discussion in Chapter IX which he says is based on the 1917 paper. He refers to Gibbs' 1899 *Nature* letter, and in Runge (1904), pages 170-180 (online) there is an example with no reference to Gibbs. Here he highlights Bôcher's (1906; 1914) generalization. He also refers to Weyl's papers 1910, online, and 1910, online.

C.N. Moore (1925) referred to Carslaw as well as a claim in *Traité d'Analyse* by E. Picard claiming Du Bois-Reymond discovered this phenomenon. online.

Carslaw 1917 begins Chapter IX with uniformly convergent series expansions for the odd function

$$f(x) = \begin{cases} -\frac{1}{4}\pi(\pi + x), & -\pi \le x \le -\frac{\pi}{2}, \\ \frac{1}{4}x, & -\frac{1}{2}\pi \le x \le \frac{\pi}{2}, \\ -\frac{1}{4}\pi(\pi - x), & \frac{\pi}{2} \le x \le \pi, \end{cases}$$

and the even function

$$f(x) = \begin{cases} \frac{1}{4}\pi(\pi + x), & -\pi \le x \le -\frac{\pi}{2}, \\ -\frac{1}{4}x, & -\frac{1}{2}\pi \le x \le 0, \\ \frac{1}{4}x, & 0 \le x \le \frac{\pi}{2}, \\ -\frac{1}{4}\pi(\pi - x), & \frac{\pi}{2} \le x \le \pi. \end{cases}$$

The respective series are

$$\sin x - \frac{1}{3^2}\sin 3x + \frac{1}{5^2}\sin 5x - \dots$$

$$\frac{\pi^2}{16} - 2\left(\frac{1}{2^2}\cos 2x + \frac{1}{6^2}\cos 6x + \frac{1}{10^2}\cos 10x + \dots\right).$$

Then when $f(x) = x$, $-\pi < x < \pi$, the Fourier series is

$$2\left(\sin x - \frac{1}{2}\sin 2x + \frac{1}{3}\sin 3x - \dots\right).$$

Gibbs in 1899 noted that the partial sums behave differently at the endpoints without proof. Extended in 1906 by Bôcher to other functions with discontinuity. Carslaw presented some of the text.

The story begins earlier with Euler and Fourier. In E189 Euler (1753) presents what we now know as a Fourier series representation. In E329 Euler (1760) he sums series of complex numbers to get things that look like Fourier series, online. There were also papers with various sums of trigonometric functions E447 and E655

Also, look into Gronwall (1912) which is here.

# A Historical Account of the Gibbs Phenomenon

The mathematical phenomenon now known as the Gibbs phenomenon has a long and curious history, beginning not with Josiah Willard Gibbs (1839-1903), after whom it is named, but with an almost forgotten English mathematician named Henry Wilbraham (1825-1883). In 1848, Wilbraham published a paper in the *Cambridge and Dublin Mathematical Journal* in which he carefully examined the behavior of Fourier series near discontinuities. He noted that when a Fourier series approximates a function with a jump discontinuity, it overshoots near the jump, and this overshoot does not vanish even as more terms are added to the series. His work included explicit calculations and recognized the limiting value of the overshoot as approximately 9% of the jump. Despite the importance of this result, Wilbraham's paper attracted little attention and was largely forgotten for nearly fifty years.

The topic reemerged in the 1890s, propelled in part by the experimental work of Albert A. Michelson (1852–1931), the American physicist best known for his measurement of the speed of light and the Michelson–Morley experiment. Michelson, deeply interested in wave phenomena, constructed a mechanical harmonic analyzer capable of summing multiple sine waves to model periodic functions. Together with his colleague F. S. Stratton, Michelson applied this apparatus to the approximation of a square wave. On October 6, 1898, he published a brief letter in *Nature* describing the device and including a striking visual image that revealed the curious overshoot near the discontinuities of the square wave. This drew immediate criticism. Just one week later, on October 13, 1898, the British mathematician A. E. H. Love (1863–1940) responded in *Nature*, suggesting that the apparent overshoot was due to experimental or mechanical imperfections, and not an intrinsic mathematical feature.

This exchange prompted further correspondence. In December 1898, *Nature* published three letters: one by Michelson defending the validity of his results, a second by Josiah Willard Gibbs (1839–1903), professor at Yale, and a third by Love. Gibbs' letter, dated December 15, 1898, was brief but profound. He explained that the overshoot was not an artifact of Michelson's machine, but rather an inherent feature of Fourier series approximating functions with jump discontinuities. Gibbs quantified the limiting height of the overshoot as approximately 0.08949 times the jump, and noted that this did not diminish even as the number of terms in the series increased. This was, in effect, a rediscovery of Wilbraham's observation, though Gibbs was unaware of the earlier work.

The discussion did not end there. In April 1899, Gibbs followed up with a longer and more technical letter in *Nature*, providing further justification and clarification of his earlier claims. Michelson responded again in May, affirming that his mechanical results matched Gibbs' theoretical explanation. Love, in turn, published a final letter in June 1899, conceding that the overshoot was indeed a genuine phenomenon intrinsic to the convergence behavior of Fourier series.

Thus, through a series of public letters in *Nature*, a vibrant and sometimes contentious discussion unfolded between experiment and theory, culminating in a deeper understanding of the behavior of Fourier series. While Michelson's experimental ingenuity brought the phenomenon into view, and Love's skepticism spurred clarification, it was Gibbs who offered the concise theoretical account that would carry his name.

In the early 20th century, other mathematicians began to adopt and expand upon Gibbs' insight. Maxime Böcher (1867–1918), a professor at Harvard and a specialist in differential equations, formalized the terminology by referring explicitly to the "Gibbs phenomenon" in his 1906 writings. This helped establish the term within the mathematical community. Horatio Scott Carslaw (1870–1954), an Australian mathematician working in Scotland, further popularized the concept through his influential texts on Fourier series and heat conduction. Meanwhile, the young German mathematician Hermann Weyl (1885–1955) contributed to a more rigorous mathematical framework for the phenomenon through his work on integral equations and harmonic analysis. Weyl's early analytical treatments clarified how the phenomenon emerged not from a failure of convergence, but from the nature of pointwise convergence near discontinuities.

Over the next few decades, the term "Gibbs phenomenon" became standard, despite Wilbraham's prior contribution. It wasn't until the mid-twentieth century that historians of mathematics rediscovered Wilbraham's original 1848 paper, leading to occasional calls for the phenomenon to be more properly attributed or at least to recognize Wilbraham's precedence. Nevertheless, the name "Gibbs phenomenon" persists, a testament to the enduring impact of Gibbs' concise and influential communication in *Nature* and the mathematical community's recognition of his broader contributions to theoretical physics.

Today, the Gibbs phenomenon is a staple of mathematical analysis and signal processing, illustrating the subtleties of pointwise convergence and the limits of Fourier approximations. It stands as a striking example of how mathematical discoveries can be overlooked, rediscovered, debated in public forums, and gradually absorbed into the fabric of scientific understanding through the efforts of many individuals across time.

# References


Bôcher, Maxime. 1906. "Introduction to the Theory of Fourier's Series." *The Annals of Mathematics* 7 (3): 81–152.

———. 1914. "On Gibb's Phenomenon." *J. Math. Berlin.*

Burkhardt, Heinrich. 1908. *Entwicklungen nach oscillirenden Functionen und Integration der Differentialgleichungen der mathematischen Physik: Bericht, erstattet der Deutschen Mathematiker-Vereinigung.* Vol. 1. [Developments according to oscillating functions and integration of the differential equations of mathematical physics: Report, submitted to the German Association of Mathematicians.] BG Teubner.

Byerly, William Elwood. 1893. "An Elementary Treatise on Fourier's Series and Spherical." *bme* 1000:1.

Carslaw, Horatio Scott. 1925a. "A historical note on Gibbs' phenomenon in Fourier's series and integrals." *Bull. Amer. Math. Soc.* 31 (8): 420–424.

———. 1917. "A trigonometrical sum and the Gibbs' phenomenon in Fourier's series." *American Journal of Mathematics* 39 (2): 185–198.

———. 1925b. "Gibbs' Phenomenon in Fourier's Integrals." *Nature* 116 (2913): 312–313.

———. 1921. *Introduction to the Theory of Fourier's Series and Integrals.* 2nd ed. Macmillan / Company, limited.

De Morgan, Augustus. 1836. *Elementary Illustrations of the Differential and Integral Calculus.* Open Court Publishing Company.

Euler, Leonhard. 1753. "De serierum determinatione seu nova methodus inveniendi terminos generales serierum." *Novi commentarii academiae scientiarum Petropolitanae,* 36–85.

———. 1760. "Subsidium calculi sinuum." *Novi Commentarii academiae scientiarum Petropolitanae,* 164–204.

Fourier, Jean Baptiste Joseph. 1808. "Mémoire sur la propagation de la chaleur dans les corps solides (extrait)." [Memoir on the Propagation of Heat in Solid Bodies]. *Nouveau Bulletin des Sciences, par la Société Philomathique de Paris* 1:112–16.

———. 1878. *The Analytic Theory of Heat, English translation by A Freeman.*

———. 1822. *Théorie analytique de la chaleur.* [Analytical Theory of Heat]. Firmin Didot.

Gibbs, Josiah Willard. 1898. "Fourier's Series." Dec. 29, *Nature* 59:200.

———. 1899. "Fourier's Series." Apr. 27, *Nature* 59:606.

Gottlieb, David, and Chi-Wang Shu. 1997. "On the Gibbs phenomenon and its resolution." *SIAM review* 39 (4): 644–668.

Gregory, David F. 1838. "Notes on Fourier's Heat." *Cambridge Math. J* 1:104–107.

Gregory, D.F., R.L. Ellis, W.T.B. Kelvin, and N.M. Ferrers. 1847. *Cambridge Mathematical Journal.* v. 2; v. 6. Macmillan, Barclay / Macmillan. https://books.google.com/books?id=zC46AAAAMAAJ.

Gronwall, Thomas Hakon. 1912. "Über die Gibbssche Erscheinung und die trigonometrischen Summen $\sin x + \frac{1}{2}\sin 2x + \cdots + \frac{1}{n}\sin nx$." *Mathematische Annalen* 72:228–243.

Hewitt, Edwin, and Robert E Hewitt. 1979. "The Gibbs-Wilbraham Phenomenon: An Episode in Fourier Analysis." *Archive for history of Exact Sciences,* 129–160.

Jerri, Abdul J. 1998. *The Gibbs Phenomenon in Fourier Analysis, Splines and Wavelet Approximations.* Vol. 446. Springer Science & Business Media.

Love, Augustus Edward Hough. 1898a. "Fourier's Series." Oct. 13, *Nature* 58:569–570.

Love, Augustus Edward Hough. 1898b. “Fourier’s Series.” Dec. 29, *Nature* 59:200–201.

———. 1899. “Fourier’s Series.” June 1, *Nature* 58:100–2.

Michelson, Albert A. 1898. “Fourier’s Series.” Oct. 6, *Nature* 58:544–545.

———. 1899. “Fourier’s Series.” May 18, *Nature* 58:52.

Michelson, Albert Abraham, and Samuel Wesley Stratton. 1898. “A New Harmonic Analyzer.” *American Journal of Science* 4 (25): 1–13.

Moore, Charles Napoleon. 1925. “Note on Gibbs’ phenomenon.” *Bulletin of the American Mathematical Society* 31 (8): 417–419.

Newman, Francis W. 1848. “On the values of a periodic function at certain limits.” *Cambridge & Dublin Math. J* 3:108–111.

Runge, Carl. 1904. *Theorie und Praxis der Reihen.* Vol. 32. GJ Göschen.

Thomson, William. 1841. “Note on a Passage in Fourier’s Heat.” *Camb. J. Math.* 3:25–27.

———. 1939. “On Fourier’s Expansion of Functions in Trigonometric Series.” Publication Date 1841. *Camb. J. Math.* 2:258–262.

Weyl, Hermann. 1910a. “Die gibbs’ sche erscheinung in der theorie der kugelfunktionen.” *Rendiconti del Circolo Matematico di Palermo (1884-1940)* 29:308–323.

———. 1910b. “Über die Gibbs’ sche Erscheinung und verwandte Konvergenzphänomene.” *Rendiconti del Circolo Matematico di Palermo (1884-1940)* 30:377–407.

Wilbraham, H. 1848. “On certain periodic functions.” *Cambridge Dublin Math J* 3:198–204.