

Instructions:

- Place your name on all of the pages.
- Do all of your work in this booklet. Do not tear off any sheets.
- Show all of your steps in the problems for full credit.
- Be clear and neat in your work. Any illegible work, or scribbling in the margins, will not be graded.
- Put a box around your answers when appropriate..
- If you need more space, you may use the back of a page and write *On back of page #* in the problem space or the attached blank sheets. **No other scratch paper is allowed.**

Try to answer as many problems as possible. Provide as much information as possible. Show sufficient work or rationale for full credit. Remember that some problems may require less work than brute force methods.

If you are stuck, or running out of time, indicate as completely as possible, the methods and steps you would take to tackle the problem. Also, indicate any relevant information that you would use.

Pace yourself – do not spend more than 15 minutes per page on your first pass.

Pay attention to the point distribution. Not all problems have the same weight.

Page	Pts	Score
1	14	
2	12	
3	15	
4	9	
Total	50	

Bonus: Evaluate $L \left[\int_0^t \sin 3u \, du \right]$.

1. (6 pts) Do the following:
 - a. Define the inverse Fourier transform, $f(t)$, of $\hat{f}(\omega)$.
 - b. Give an integral expression for the inverse Laplace transform of $F(s)$.
 - c. What is the Fourier version of the convolution of two functions?

2. (2 pts) Find the Fourier transform in simplest form of the function $f(x) = \pi[\delta(x-a) - \delta(x+a)]$.

3. (3 pts) Prove that $F\left[\frac{df}{dt}\right] = -ik\hat{f}(k)$ and state any conditions needed to make this true.

4. (3 pts) Evaluate: $\int \delta(x^2 - 5x + 6)x^2 dx$.

5. (3 pts) Find the Fourier transform of $f(t) = e^{-2(t-1)}H(t-1)$.
6. (6 pts) Let $f(t) = t^2$ and $g(t) = \sin 3t$.
- Compute $(f * g)(t)$ in the Laplace transform sense.

 - Find $L[f * g]$.
7. (3 pts) Solve $y'' + 2y' + y = \delta(t) - \delta(t-2)$, for $y(0) = 1$, $y'(0) = 0$.

8. (6 pts) Compute the Laplace transforms:

a. $f(t) = (t^2 + 4t + 2)e^{3t}$.

b. $f(t) = \begin{cases} 2, & 0 \leq t \leq 4, \\ t^2, & t \geq 4. \end{cases}$

9. (9 pts) Find the inverse Laplace transforms:

a. $L^{-1} \left[\frac{1}{(s+3)(s-4)} \right]$.

b. $L^{-1} \left[\frac{2s-3}{s^2+4s+1} \right]$.

c. $L^{-1} \left[\frac{3se^{-s}}{s^2+16} \right]$.

10. (3 pts) Find the inverse Fourier transform of $F(\omega) = \frac{\omega}{\omega^2 + 9}$.

11. (3 pts) Find the Fourier transform of $f(x) = xe^{-x^2}$.

12. (3 pts) Let $F(s) = \frac{2s-1}{(s-1)(s+5)}$. Find $f(t)$ using the Bromwich integral.

MAT 367 Exam III

Name _____

Extra Space