MAT 367 Exam I **Name ________________________**

Instructions:

- Place your name on all of the pages.
- Do all of your work in this booklet. Do not tear off any sheets.
- Show all of your steps in the problems for full credit.
- Be clear and neat in your work. Any illegible work, or scribbling in the margins, will not be graded.
- Put a box around your answers when appropriate..
- If you need more space, you may use the back of a page and write *On back of page #* in the problem space or the attached blank sheets. **No other scratch paper is allowed.**

Try to answer as many problems as possible. Provide as much information as possible. Show sufficient work or rationale for full credit. Remember that some problems may require less work than brute force methods.

If you are stuck, or running out of time, indicate as completely as possible, the methods and steps you would take to tackle the problem. Also, indicate any relevant information that you would use.

Pace yourself – do not spend more than 15 minutes per page on your first pass.

Pay attention to the point distribution. Not all problems have the same weight.

Page	Pts	Score
1	20	
2	18	
3	13	
4	14	
Total	65	

Bonus: Do only one.

a) Sum the series $\sum_{n=2}^{\infty} \frac{2}{n(n+1)}$.

b) Expand $\sqrt{\frac{1+x}{1-x}}$ up to and including x^2 terms. Setting $x = \frac{1}{10}$, show $\sqrt{11} \approx \frac{663}{200}$.

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1. (9 pts) Find the limit of each sequence $\{a_n\}$ as $n \to \infty$, showing your work.

   a. $a_n = (2n)^{1/n} \dfrac{n^2 + 4n}{5n^2 - 3\sqrt{n^4 + 2n}}$.

   b. $a_n = \dfrac{(2n^2 + 3n - 5)\ln(n^2)}{n^3}$.

   c. $a_n = \left(\dfrac{2n}{2n+5}\right)^n$.

2. (3 pts) Find the sum of the series $\displaystyle\sum_{n=0}^{\infty} (-1)^n \frac{3^n}{2^{2n+1}}$

3. (5 pts) Do the following:

   a. In the expansion of $\left(2x - \dfrac{3}{x}\right)^7$ what is the numerical coefficient of x?

   b. Use Euler's formula to write $e^{i\pi/6}$ exactly, using radicals, in the form $a + bi$.

4. (3 pts) Determine the order of $f(x) = \ln(1+x)^2 - 2\sin x + x^2$ as $x \to 0$.

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5. (9 pts) Determine if the following series diverge, converge absolutely, or converge conditionally. Show how you reached your conclusion noting the test used.

   a. $\sum_{n=1}^{\infty} \frac{2n-3}{2n}$.

   b. $\sum_{n=0}^{\infty} (-1)^n \frac{n+5}{n\sqrt{n+3}}$.

   c. $\sum_{n=1}^{\infty} \frac{2^n}{n!} \cos n\pi$.

6. (3 pts) Compute $\sqrt{1.01} = \sqrt{1+0.01}$ to six decimal places without a calculator.

7. (6 pts) Consider the series $\sum_{n=0}^{\infty} 3\left(\frac{x+1}{4}\right)^n$.

   a. Find the radius and interval of convergence.

   b. In the interval of convergence, what is the sum of this series (in simplest form)?

8. (9 pts) Find the Taylor series expansion for the following about the given point. If you cannot write the general form, then provide the first three nonzero terms of the expansion.

   a. $f(x) = e^{(x-2)/3},\ x = 2.$

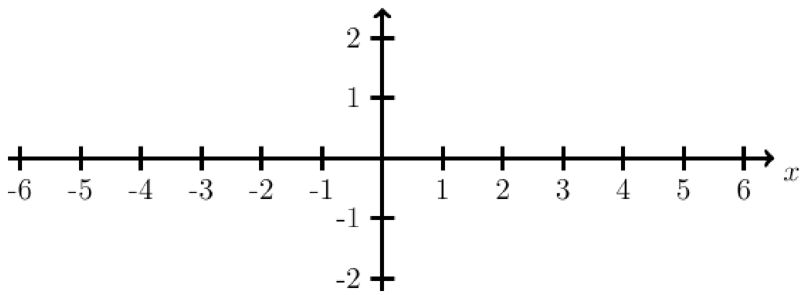
   b. $f(x) = (1-x)^{3/2},\ x = 0.$

   c. $f(x) = \dfrac{1}{x+3},\ x = 1.$

9. (4 pts) Determine the nonzero Fourier coefficients for $f(x) = 5 + 3\cos 3x - 4\sin^2 2x,\ x \in [-\pi, \pi]$ without doing any integration.

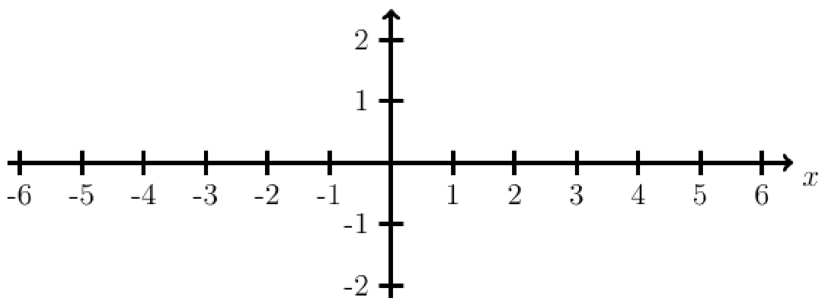
10. (14 pts) Consider $f(x) = x(2-x),\ x \in [0,2]$.

a. Sketch what you would expect for the Fourier trigonometric series on the interval $-6 < x < 6$.



b. What is the period of this periodic extension? ______

c. Sketch the odd periodic extension of this function on the interval $-6 < x < 6$.



d. What is the period of the odd periodic extension? ______

e. Find the Fourier series representation of $f(x)$.

f. Find the Fourier sine series of the above $f(x)$.

Extra Space