MAT 367 Final Exam **Name ________________________**

Instructions:

- Place your name on all of the pages.
- Do all of your work in this booklet. Do not tear off any sheets.
- Show all of your steps in the problems for full credit.
- Be clear and neat in your work. Any illegible work, or scribbling in the margins, will not be graded.
- Put a box around your answers when appropriate..
- If you need more space, you may use the back of a page and write *On back of page #* in the problem space or the attached blank sheet. **No other scratch paper is allowed.**

Try to answer as many problems as possible. Provide as much information as possible. Show sufficient work or rationale for full credit. Remember that some problems may require less work than brute force methods.

If you are stuck, or running out of time, indicate as completely as possible, the methods and steps you would take to tackle the problem. Also, indicate any relevant information that you would use. Do not spend too much time on one problem. **Pace yourself.**

Pay attention to the point distribution. Not all problems have the same weight.

Page	**Pts**	**Score**
1	**11**	
2	**21**	
3	**21**	
4	**11**	
5	**12**	
6	**15**	
7	**9**	
Total	**100**	

Have a good summer!

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1. (5 pts) Finish the following statements:
   a. The Fourier transform of a box function is a ____________________.

   b. The Laplace transform of $\int_0^t f(\tau)\, d\tau$ is _______________________ .

   c. The inverse Fourier transform of a product of Fourier transforms is

      ____.

   d. The function $f(t)=\begin{cases} 1, & 0 \le t \le 1 \\ e^{-2t}, & \text{t>1,} \end{cases}$ is given in terms of Heaviside

      functions as ____.

   e. The Fourier sine series of $f(x)$ on $[0,2]$ is given by

      ____.

2. (3 pts) Find the inverse Fourier transform, $f(x)$, of $\hat{f}(k)=\dfrac{1}{1-ik}$.

3. (3 pts) Evaluate $\int_{-\infty}^{\infty} \sin x\, e^{i(k-2)x} dx$ in terms of delta functions. [Hint: First write the integrand in terms of exponentials.]

4. (7 pts) Consider the sequence of functions defined by $f_n(x) = \dfrac{n}{1+4n^2(x-1)^2}$.

   a. Sketch $f_n(x)$ and label the location and height of the peak.

   b. Find the area under each function in the sequence.

   c. Based on these results, $\lim_{n\to\infty} f_n(x) =$

5. (6 pts) Evaluate the following:

   a. $\displaystyle\int_{-\infty}^{\infty} (x^2+3x-10)\delta(x+2)\,dx =$

   b. $\displaystyle\int_{0}^{\infty} \delta(x^2+3x-10)(x+2)\,dx =$

6. (8 pts) Let $F(s) = \dfrac{1}{s(s+9)}$.

   a. Use the Bromwich integral to find the inverse Laplace transform.

   b. Use the Convolution Theorem to find the inverse Laplace transform.

7. (3 pts) Prove $L^{-1}\left[e^{-as}F(s)\right] = f(t-a)H(t-a)$.

8. (9 pts) Find the Laplace transforms:
   a. $L[e^{-3t}\sin 2t] =$

   b. $L[t\sinh t] =$

   c. $L[H(t-\frac{\pi}{2})\sin t] =$

9. (6 pts) Find the inverse Laplace transforms:
   a. $L^{-1}\left[\frac{s}{(s-2)(s+3)}\right] =$

   b. $L^{-1}\left[\frac{s-1}{s^2+4s+20}\right] =$

10. (3 pts) Evaluate $\int_{-\infty}^{\infty} e^{-9x^2}\,dx.$

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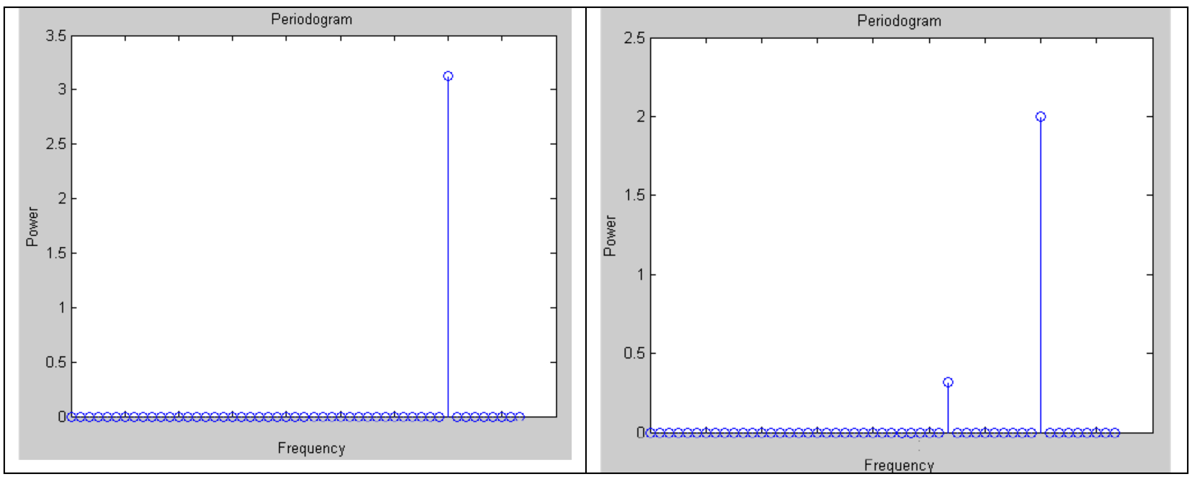
11. (3 pts) Prove that $F\left[Ш\left(x\right)\right] = 2\pi \text{comb}_{2\pi}(\omega)$ using $\sum_{n=-\infty}^{\infty} e^{ina\omega} = \frac{2\pi}{a}\text{comb}_{\frac{2\pi}{a}}(\omega)$.

12. (8 pts) Consider the spectra below. In each case the data was sampled at 100 points for 12 seconds. [Do not forget the units in your answers.]



a. What is the sampling rate, f_s? __________
b. What is Δf? __________
c. What is the maximum frequency in the plots? __________
d. Determine the frequency of the spike in the first plot. __________
e. In the first figure, determine the possible signal,

$y(t) =$ ______________________

f. In the second spectrum there are two frequencies. The tall spike is an aliased frequency. What the correct frequency would be displayed if the data were sampled using 150 points? __________

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13. (12 pts) Consider the function $f(t) = t$ for $t \in [0, \pi]$.

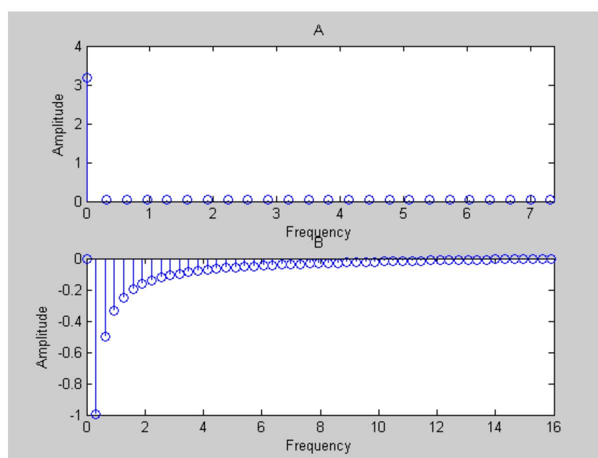
a. Find the Fourier transform of $f(t)$, assuming that $f(t) = 0$ outside the given interval.

b. Find the Fourier coefficients in the Fourier series expansion,

$$f(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos 2nt + b_n \sin 2nt.$$

c. Noting that $\omega_n = 2\pi f_n = 2n,$ find a relation between the Fourier transform, $\hat{f}(\omega_n)$, and the Fourier coefficients in parts a and b.

d. The DFT implementation in Matlab for $f(t)$ on $[0, \pi]$ using $N = 100$ points is given in the plot. How do the values in the plot relate to your answers above? If you did not do the above parts, what does this graph tell you that you should have gotten?



14. (4 pts) Find the Fourier transform of $f(x) = e^{-|x|}$ and simplify.

15. (4 pts) Prove that $\sum_{n=0}^{N-1} e^{2\pi ikn/N} = \begin{cases} 0, & k = 1, \ldots, N-1 \\ N, & k = 0, N \end{cases}$ for *k* an integer.

16. (4 pts) Use Laplace transforms to solve $y' + 3y = 2\delta(t-1)$ subject to the initial condition $y(0) = 1$.

17. (3 pts) Evaluate $\dfrac{\Gamma\left(\frac{16}{3}\right)}{5!!\Gamma\left(\frac{10}{3}\right)}$.

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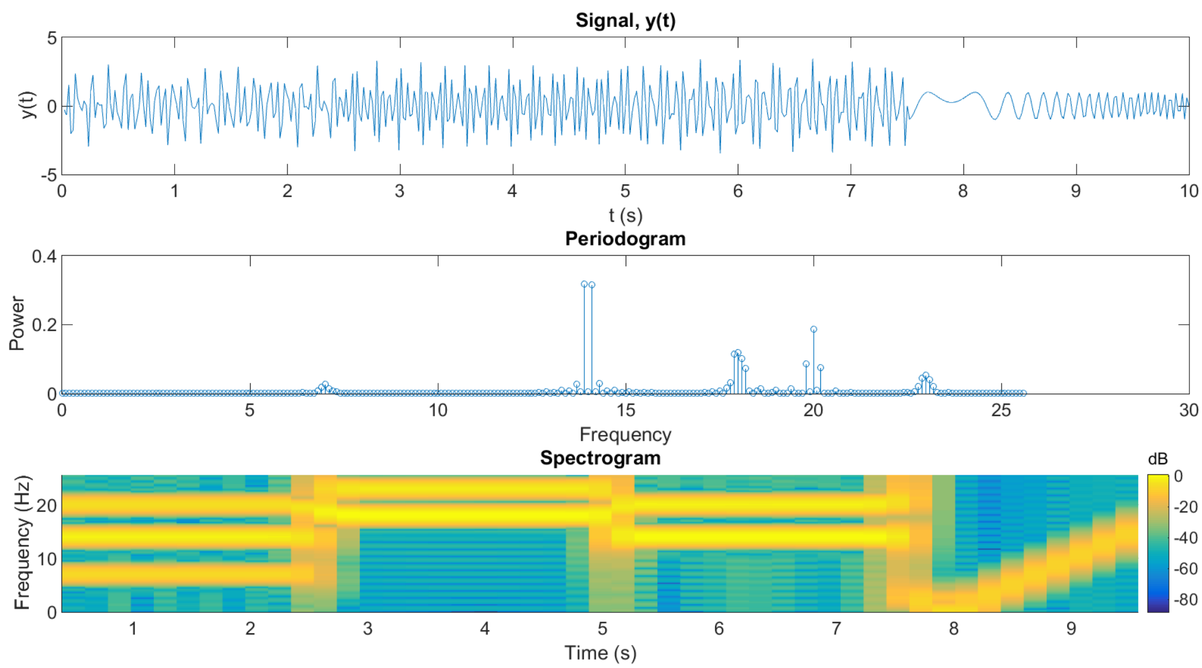
18. (4 pts) Evaluate $\oint\limits_{|z-4|=2} \frac{z^2+1}{z(z-4)^2}\,dz$

19. (5 pts) From the below plots what can you say about the signal? Be specific about the frequency value and types of signal present.



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Extra Space