MAT 367 Final Exam **Name _________________________**

Instructions:

- Place your name on all of the pages.
- Do all of your work in this booklet. Do not tear off any sheets.
- Show all of your steps in the problems for full credit.
- Be clear and neat in your work. Any illegible work, or scribbling in the margins, will not be graded.
- Put a box around your answers when appropriate..
- If you need more space, you may use the back of a page and write *On back of page #* in the problem space or the attached blank sheet. **No other scratch paper is allowed.**

Try to answer as many problems as possible. Provide as much information as possible. Show sufficient work or rationale for full credit. Remember that some problems may require less work than brute force methods.

If you are stuck, or running out of time, indicate as completely as possible, the methods and steps you would take to tackle the problem. Also, indicate any relevant information that you would use. Do not spend too much time on one problem. **Pace yourself.**

Pay attention to the point distribution. Not all problems have the same weight.

Page	Pts	Score
1	13	
2	19	
3	22	
4	14	
5	11	
6	10	
Total	90	

Have a good summer!

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1. (10 pts) Complete the following statements:
   a. The Gamma function is the generalization of the ____________________

      ____.

   b. The Fourier transform of a convolution is the _______________________

      ____.

   c. The function $f(t) = \begin{cases} \cos t, & 0 \le t \le \pi \\ 0, & \text{otherwise,} \end{cases}$ can be written in terms of Heaviside functions as

   d. The Fourier transform of the inverse Fourier transform of $\hat{f}(k)$ is _____

      ____

   e. The second shift theorem for Fourier transforms is given by

   f. In sampling theory we introduced the function $\text{comb}_a(t)$. This is defined as $\text{comb}_a(t) =$ ____________________ and it is called the _____________ function.

2. (3 pts) Evaluate $\int_{-\infty}^{\infty} \sin 3x\, e^{ikx} dx$ in terms of delta functions. [Hint: First write the integrand in terms of exponentials.]

3. (7 pts) Consider the sequence of functions defined by $f_n(x) = \dfrac{n}{1+n^2x^2}$.

   a. Sketch $f_n(x)$ and label the location and height of the peak.

   b. Find the area under each function in the sequence.

   c. Based on these results, find $\lim\limits_{n\to\infty} f_n(x) =$

4. (3 pts) Evaluate the following: $\displaystyle\int_{-\infty}^{\infty} \delta(x^2-9)(x+2)\,dx.$

5. (6 pts) Let $F(s) = \dfrac{1}{s(s+1)}$.

   a. Use the Bromwich integral to find the inverse Laplace transform.

   b. Use the Convolution Theorem to find the inverse Laplace transform.

6. (3 pts) Prove $L^{-1}\left[e^{-as}F(s)\right] = f(t-a)H(t-a).$

7. (9 pts) Find the Laplace transforms:
   a. $L[e^{-2t}\sin 3t] =$

   b. $L[t\cosh t] =$

   c. $L[H(t-\pi)\cos t] =$

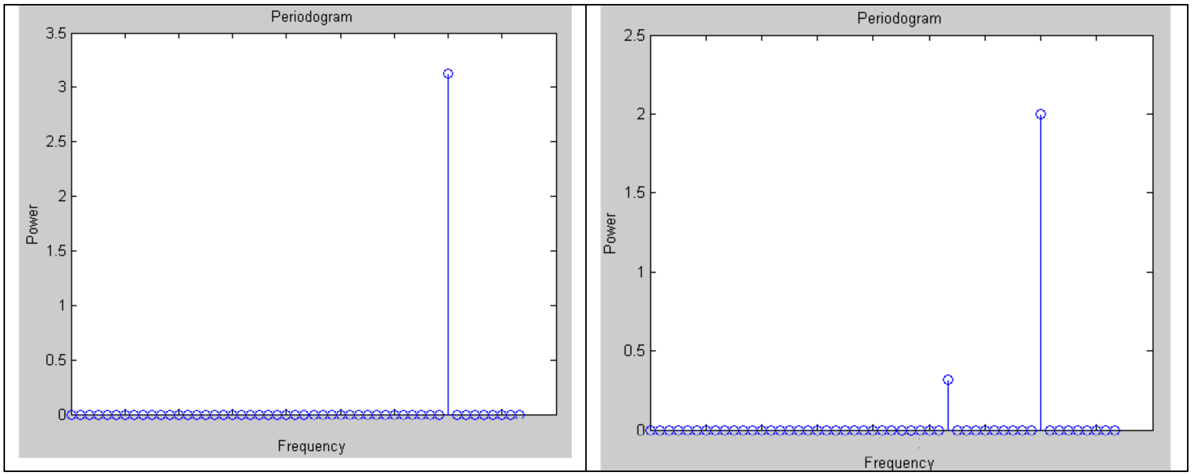
8. (6 pts) Find the inverse Laplace transforms:
   a. $L^{-1}\left[\dfrac{s}{(s-2)(s+3)}\right] =$

   b. $L^{-1}\left[\dfrac{s-1}{s^2+4s+20}\right] =$

9. (4 pts) Use Laplace transforms to solve $y'+3y = 2\delta(t-1)$ subject to the initial condition $y(0)=1$.

10. (3 pts) Prove that $F\left[\dfrac{d^2 f}{dx^2}\right] = -k^2\hat{f}(k)$. What assumptions about $f(x)$ are needed?

11. (10 pts) Consider the spectra below. In each case the data was sampled at 100 points for 12 seconds. [Use units in your answers where appropriate.]



a. What is the sampling rate, f_s? __________

b. What is Δf? __________

c. What is the maximum frequency in the plots? __________

d. Determine the frequencies of the three spikes in the plots.

__________, __________, __________.

e. In the first figure, determine the possible signal (function) giving rise to this spectrum.

$y(t) =$ ______________________

f. In the second spectrum there are two frequencies. The tall spike is an aliased frequency. The data should have been sampled with 150 points. What is the correct frequency that would be displayed using 150 points?

____

12. (4 pts) Find the Laurent series expansion of $f(z) = \dfrac{\cos 2z}{z^3}$.

What is the residue of $f(z)$ at $z = 0$?

13. (3 pts) Evaluate $\oint_{|z-1|=5} \frac{z^2+3}{z^2(z-3)}\,dz$

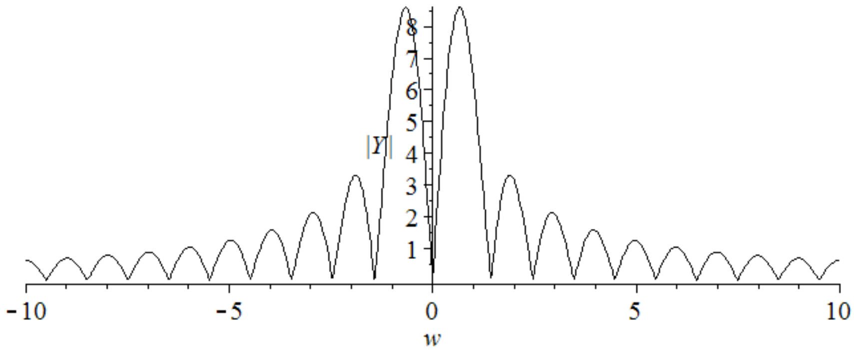
14. (8 pts) Consider the finite wave train $f(t)=\begin{cases} t, & -\pi \le t \le \pi. \\ 0, & \text{otherwise.} \end{cases}$

a. Find the Fourier transform $\hat{f}(\omega)$ of $f(t)$ and simplify if possible.

b. Find the Fourier coefficients in the Fourier series expansion of

$$f(t)=t,\ \ -\pi \le t \le \pi,\quad f(t)=\frac{a_0}{2}+\sum_{n=1}^{\infty} a_n \cos nt + b_n \sin nt.$$

15. (4 pts) A plot of $\hat{f}(\omega)$ from 14 a of the last problem is shown below.



a. Use this plot to explain how the Fourier coefficients in 14 b of the last problem are related to the Fourier transform from 14 a.

b. Describe how this plot would change if $f(t)=\begin{cases} t, & -2\pi \le t \le 2\pi, \\ 0, & \text{otherwise.} \end{cases}$

16. (3 pts) Determine the value of $\sum_{n=0}^{N-1} \cos \frac{2\pi nk}{N}$ for $k = 0,1,\ldots,N$.

17. (3 pts) Find the Fourier transform of $f(x) = e^{-2|x|}$ and simplify.

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Extra Space