Topics for MAT 367 Exam

I. Fourier Series

a. Trigonometric on *[0,L]*

i. $f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\frac{2n\pi x}{L} + b_n \sin\frac{2n\pi x}{L}$

ii. $a_n = \frac{2}{L}\int_0^L f(x)\cos\frac{2n\pi x}{L}\,dx$

$b_n = \frac{2}{L}\int_0^L f(x)\sin\frac{2n\pi x}{L}\,dx$

b. Sine on *[0,L]*

i. $f(x) = \sum_{n=1}^{\infty} b_n \sin\frac{n\pi x}{L}$

ii. $b_n = \frac{2}{L}\int_0^L f(x)\sin\frac{n\pi x}{L}\,dx$

c. Cosine on *[0,L]*

i. $f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\frac{n\pi x}{L}$

ii. $a_n = \frac{2}{L}\int_0^L f(x)\cos\frac{n\pi x}{L}\,dx$

d. Even and Odd Periodic Extensions of Functions

e. Know your integrals! Ex $\int_a^b x^k \cos(\frac{n\pi x}{L})\,dx$

II. Gamma functions, factorials, and other useful operations.

III. Complex Integration

a. Cauchy's Theorem

b. Cauchy Integral Theorem

c. Computing Residues

i. $\text{Res}[f(z); z = z_0] = \lim_{z\to z_0}(z - z_0) f(z)$ - simple poles

i. $\text{Res}[f(z); z = z_0] = \lim_{z\to z_0}\frac{1}{(k-1)!}\frac{d^{k-1}}{dz^{k-1}}[(z - z_0)^k f(z)]$ - poles of order

b. Residue Theorem $\int_C f(z)\,dz = 2\pi i \sum_{\text{Poles inside C}}$ Residues

IV. Fourier Transform

a. $\hat{f}(\omega) \equiv F[f(t)] \equiv \int_{-\infty}^{\infty} f(t)e^{i\omega t}\,dt.$

b. $f(t) \equiv F^{-1}[\hat{f}(\omega)] \equiv \frac{1}{2\pi}\int_{-\infty}^{\infty} \hat{f}(\omega)e^{-i\omega t}\,d\omega.$

c. Properties – Transforms of derivatives, Shifting properties

d. Gaussian Integrals, Transform of a Gaussian

e. Convolution Theorem $F^{-1}\left[F(\omega)G(\omega)\right] = (f * g)(x) \equiv \int_{-\infty}^{\infty} f(\xi)g(x-\xi)\,d\xi$

V. Dirac Delta Function

a. Definition, limits of sequences of functions

b. $\int_{-\infty}^{\infty} e^{ikx}dk = 2\pi\delta(x)$

c. $\int_{-\infty}^{\infty} \delta(x-a) f(x) dx = f(a)$

d. $\int_{-\infty}^{\infty} \delta(f(x)) g(x) dx = \int_{-\infty}^{\infty} \sum_{i=1}^{n} \frac{\delta(x-x_i)}{|f'(x_i)|} g(x) dx$ for simple roots.

VI. Laplace Transform

a. Definition $Y(s) \equiv L\{y(t)\} \equiv \int_0^{\infty} y(t) e^{-st}\, dt.$

b. Particular Functions $t^n, e^{at}, \sin at, \cos at, H(t-a), \delta(t-a), \ldots$

$f(t)$	$F(s)$	$f(t)$	$F(s)$
c	$\frac{c}{s}$	e^{at}	$\frac{1}{s-a}, s < a.$
t^n	$\frac{n!}{s^{n+1}}, s > 0.$	$t^n e^{at}$	$\frac{n!}{(s-a)^{n+1}}.$
$\sin \omega t$	$\frac{\omega}{s^2+\omega^2}$	$e^{at} \sin \omega t$	$\frac{\omega}{(s-a)^2+\omega^2}$
$\cos \omega t$	$\frac{s}{s^2+\omega^2}$	$e^{at} \cos \omega t$	$\frac{s-a}{(s-a)^2+\omega^2}$
$\sinh at$	$\frac{a}{s^2-a^2}$	$\cosh at$	$\frac{s}{s^2-a^2}$
$H(t-a)$	$\frac{e^{-as}}{s}, s > 0$	$\delta(t-a)$	$e^{-as},\ a \geq 0, s > 0.$

c. Properties

$L\{af(t)+bg(t)\} = aF(s)+bG(s)$.

$L\left\{\frac{dy}{dt}\right\} = sY(s) - y(0)$.

$L\left\{\frac{d^2y}{dt^2}\right\} = s^2Y(s) - sy(0) - y'(0)$.

$L\left\{e^{at} y(t)\right\} = Y(s-a)$.

$L\left\{H(t-a) y(t-a)\right\} = e^{-as} Y(s)$.

$L\{tf(t)\} = -\frac{d}{ds} F(s)$.

d. Convolution $(f * g)(t) = \int_0^t f(t-u) g(u)\, du.$

e. Bromwich Integral $f(t) = \frac{1}{2\pi i} \int_{c-i\infty}^{c+i\infty} F(s) e^{st} ds$

VII. Discrete Transform

a. Relationship between Fourier series, Fourier transform, discrete Fourier transform

b. Discrete Orthogonality

c. Effects of Sampling – aliasing, reading spectral plots, what you learned from the project.