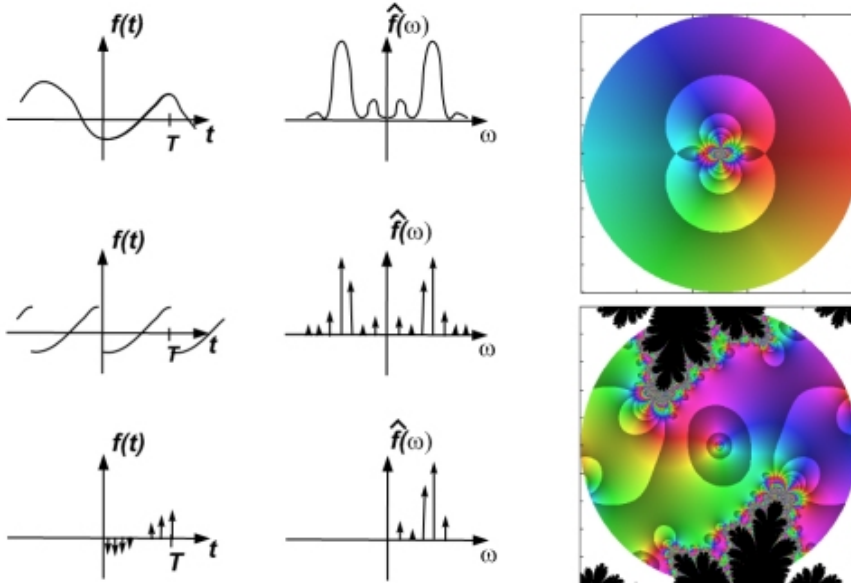
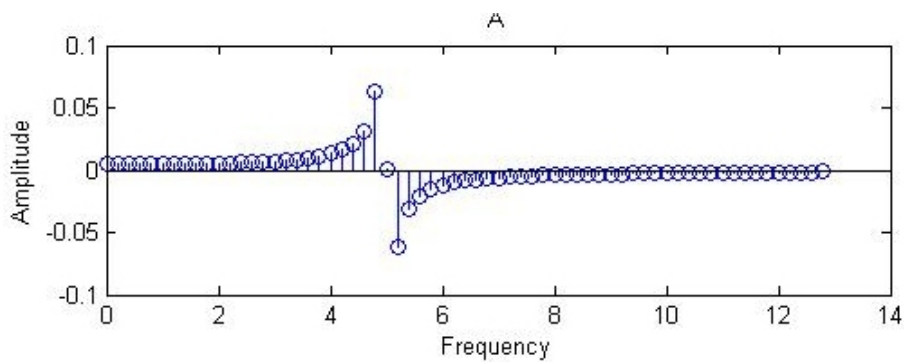


RUSSELL L. HERMAN

AN INTRODUCTION TO FOURIER AND COMPLEX ANALYSIS WITH APPLICATIONS TO THE SPECTRAL ANALYSIS OF SIGNALS



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*Dedicated to those students who have endured
the various editions of AN INTRODUCTION
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OF SIGNALS.*