

**An Introduction to Fourier and Complex
Analysis with Applications to the
Spectral Analysis of Signals**

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