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**An Introduction to Fourier  
and Complex Analysis  
with Applications to  
the Spectral Analysis of  
Signals**

Monograph

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