MAT 365 Final Exam **Name _______________________________**

Instructions:

- Place your name on all of the pages.
- Do all of your work in this booklet. Do not tear off any sheets.
- Show all of your steps in the problems for full credit.
- Be clear and neat in your work. Any illegible work, or scribbling in the margins, will not be graded.
- Put a box around your answers when appropriate..
- If you need more space, you may use the back of a page and write ***On back of page #*** in the problem space. There is also an **extra page** attached.

Try to answer as many problems as possible. Provide as much information as possible, such as sketches, etc. Show all work for full credit. Do not worry if you did not finish the exam. Do your best.

If you are stuck, or running out of time, indicate as completely as possible, the methods and steps you would take to tackle the problem. Also, indicate any relevant information that you would use. Do not spend too much time on one problem. **Pace yourself.**

Page	**Pts**	**Score**
1	**20**	
2	**15**	
3	**15**	
4	**17**	
5	**8**	
Total	**75**	

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1. (3 pts) Determine the derivative (Jacobian matrix) of $f(x,y)=(e^{x+y}+y,xy^2)$ at $(1,-1)$.

2. (6 pts) Find and classify the stationary points of $f(x,y)=x^2-3xy+5x-2y+6y^2+8$.

3. (6 pts) Consider the potential function $\phi(x,y)=2xy^{3/2}$.
   a. What is the corresponding force field?

   b. Determine the work done by this force in moving a particle from (1,1) to (2,4).

4. (5 pts) Prove $\int_R (\nabla f)\cdot \mathbf{G}\, dV = \int_{\partial R} f\,\mathbf{G}\cdot\mathbf{n}\, d\sigma - \int_R f\nabla\cdot\mathbf{G}\, dV$.

5. (9 pts) Consider the system of equations $\begin{aligned} xu + yvu^2 &= 2 \\ xu^3 + y^2v^4 &= 2. \end{aligned}$

   a. Write the corresponding linear system of equations relating the differentials *dx, dy, du,* and *dv* near (1,1,1,1).

   b. Use your system to compute

      i. $\left(\dfrac{\partial u}{\partial x}\right)_y$ at (1,1,1,1) [i.e., for $u = u(x, y)$].

      ii. $\left(\dfrac{\partial u}{\partial x}\right)_v$ at (1,1,1,1) [i.e., for $u = u(x, v)$].

6. (6 pts) Evaluate the surface integral $\int_S x\,dydz + y\,dxdy$ over the parametrized surface defined by $x = u + v, y = u^2 - v^2, z = uv,$ for $0 \le u, v \le 1$.

7. (5 pts) Consider the form $\omega = y^2\,dx + (2xy + z^2)\,dy + 2yz\,dz$
   a. Determine if ω is closed.

   b. Find the antidifferential F such that $\omega = dF$.

8. (6 pts) Consider the vector field $\mathbf{F}(x, y) = (xy, y^2)$.
   a. Compute $\nabla \bullet \mathbf{F}$

   b. Compute $\nabla \times \mathbf{F}$

   c. Does $\int_{\mathbf{c}} \mathbf{F} \bullet d\mathbf{r}$ depend on the path **c** for this vector field? Explain.

9. (4 pts) A piece of fruit is sitting on a table in a room and at each point in the room $g(x, y, z) = e^{-(x+y)^2} + z^2(x + y)$ gives the strength of the odor of the fruit. Furthermore, suppose that a bug, which maintains a speed of 2 ft/s, always flies in the direction in which the fruit odor increases fastest. What is the velocity vector of the bug when it is at the position (2,−2, 1)?

10. (12 pts) Evaluate the following integrals <u>indicating the name of the special theorem used</u>.

    a. $\int_S x^2 y\, dydz + 3y^2\, dzdx - 2xz^2\, dxdy$ for S the surface of the unit cube.
    Thm _______________

    b. $\int_C y\, dx + (2x - z)\, dy + (z - x)\, dz$ where C is the intersection of
    $x^2 + y^2 + z^2 = 4$ and $z = 1$. Thm _______________

    c. $\int_C (x^2 + y)\, dx - (3x + y^2)\, dy$ for C the ellipse $x^2 + 4y^2 = 4$.
    Thm _______________

11. (5 pts) Find the extrema of $f(x, y) = 3x + 2y$ subject to the constraint $2x^2 + 3y^2 = 3$.

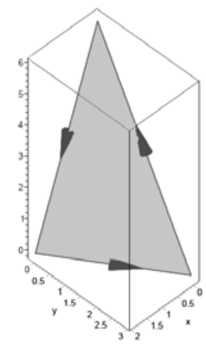
12. (8 pts) **Do <u>only one</u> of the following (a or b). Let** $I = \int_C \mathbf{F} \cdot d\mathbf{r}$

a. Verify Stokes' Theorem to compute I around the boundary of the upper half of the unit sphere for $\mathbf{F} = xy^2\mathbf{i} + y^3\mathbf{j} + y^2z\mathbf{k}$. [Namely, compute both the line integral and surface integral in Stokes' Theorem and show you get the same result.]
   i. Compute I directly. [Note: $z = 0$.]
   ii. For the surface integral, use the parametrization

$$(x, y, z) = (\sin u \cos v, \sin u \sin v, \cos u),\ \ 0 \le u \le \frac{\pi}{2},\ \ 0 \le v \le 2\pi.$$

b. Find the circulation I along C which is the boundary of the triangle $T = [(2,0,0),(0,3,0),(0,0,6)]$ which is oriented as shown for the vector field

$$\mathbf{F} = (4xy + xz)\mathbf{i} + (xy - yz)\mathbf{j} + (z^2 - xz)\mathbf{k}.$$



Which problem did you select? _______

Extra Space