1

Introduction to Simulink

There are several computer packages for finding solutions of differential equations, such as Maple, Mathematica, Maxima, MATLAB, etc. These systems provide both symbolic and numeric approaches to finding solutions. They often require a bit of coding. However, there are graphical environments for solving problems, including differential equations. One such environment is Simulink, which is closely connected to MATLAB. In these notes we will first lead the reader through Simulink examples of solutions of first and second order differential equations usually encountered in a differential equations course. We will then look at examples of more complicated systems.

Most of these models were created using Version 2015. Some changes in Versions 2017-2020 are noted.

Examples of MATLAB solutions of differential equations will also be provided.

1.1 Solving an ODE

Simulink is a graphical environment for designing simulations of systems. As an example, we will use Simulink to solve the first order differential equation (ODE)

$$\frac{dx}{dt} = 2\sin 3t - 4x. \tag{1.1}$$

We will also need an initial condition of the form $x(t_0) = x_0$ at $t = t_0$. For this problem we will let $x(0) = 0$.

We can solve Equation (1.1) by integrating $\frac{dx}{dt}$ to formally obtain

$$x(t) = \int (2\sin 3t - 4x(t))\, dt.$$

We will view this as a system in which the input, $x' = 2\sin 3t - 4x$, is fed into an integrator and the output will be $x(t)$. Generally, we have

$$x(t) = \int x'(t)\, dt.$$

This process is depicted in Figure 1.1.

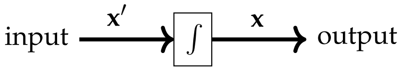


Figure 1.1: Schematic for a general system in which the block takes the input and produces an output.

In order to carry this out, we separately insert the terms $2\sin 3t$ and $-4x$ into the integration procedure. Since we do not know $-4x$, we take the output from the integrator, multiply it by 4, and subtract that from $2\sin 3t$. This combined set of terms is then feed back into the integrator. This is shown schematically in Figure 1.2.

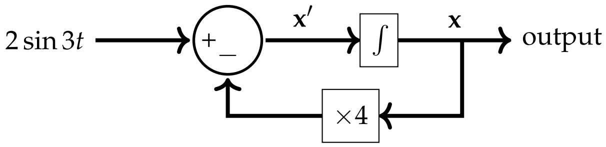


Figure 1.2: Schematic for solving $x' = 2\sin 3t - 4x$. The terms $2\sin 3t$ and $4x$ are fed into the integrator and x is output.

When you have access to Simulink and MATLAB, you can start MATLAB by typing **simulink** on the command line to bring up Simulink. Alternatively, you can select Simulink on the MATLAB icon bar to launch Simulink. Starting in 2017 Simulink opens with a start screen in which there are several selections as shown in Figure 1.3.[1] Pick the Blank Model to begin a new model or select a recently opened model. Then, you will be in the Simulink workspace [see Figure 1.4].

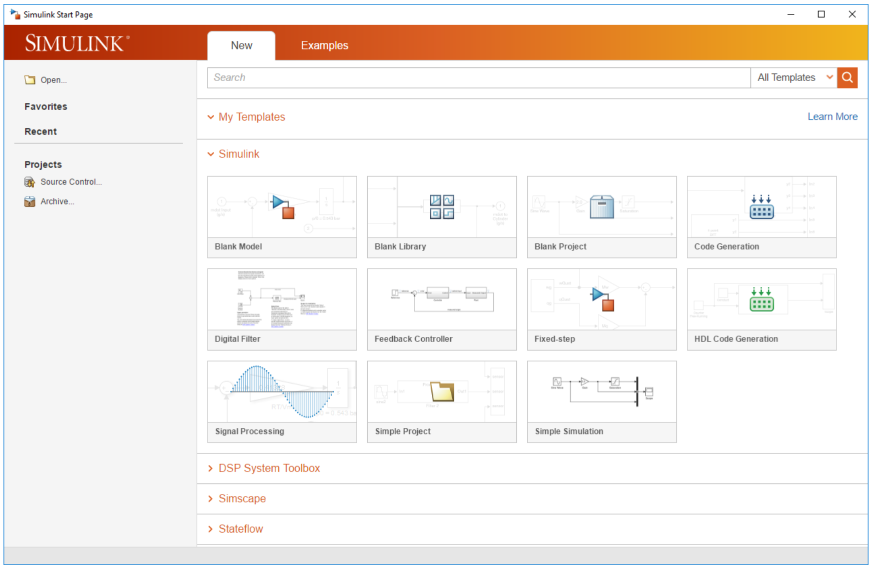
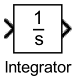


Figure 1.3: The Simulink Start screen. Pick the Blank Model to begin a new model or select a recently opened model.

From the workspace you can open the Simulink Library Browser as shown in Figure 1.5[2]. Next, click the yellow plus to bring up a new model. We build models by dragging and connecting the needed components, or blocks, from groups such as the Continuous, Math Operations, Sinks, or Sources.

Now we can create the model for simulating Equation (1.1) in Simulink as described in Figure schema2 using Simulink blocks and a differential equation (ODE) solver. In the background Simulink uses one of MATLAB's ODE solvers, numerical routines for solving first order differential equations, such as **ode45**. This system uses the **Integrator** block[3]  to integrate $\dfrac{dx}{dt}$, producing $x(t)$.

[1] In earlier versions the Simulink Library Browser in Figure 1.5 would appear.

[2]

[3] The notation on the **Integrator** block is related to the Laplace transform

$$\mathcal{L}\left[\int_0^t f(\tau)\,d\tau\right] = \frac{1}{s}F(s),$$

where $F(s)$ is the Laplace transform of $f(t)$.

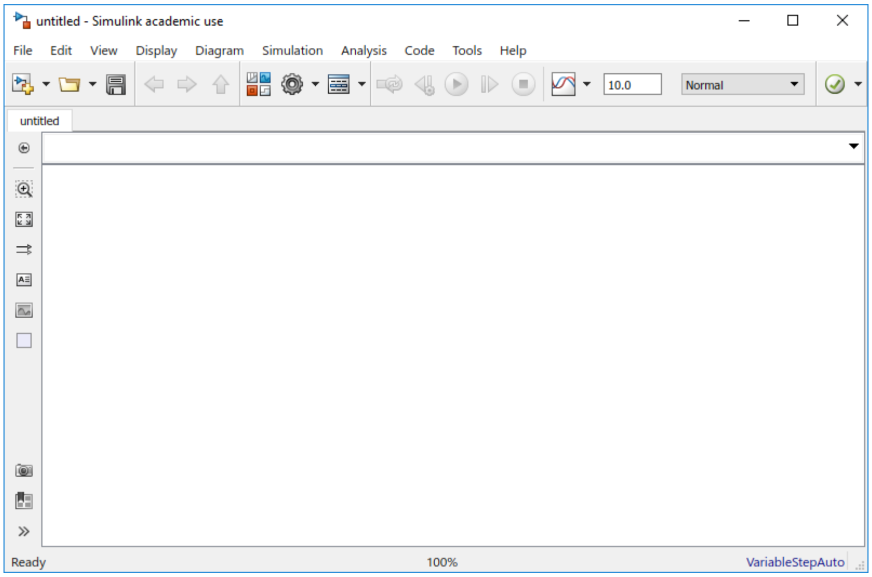


Figure 1.4: A blank model in Simulink.

The input for the **Integrator** is the right side of the differential Equation (1.1), $2\sin 3t - 4x$. The sine function can be provided by using the **Sine Wave** block, whose parameters are set in the **Sine Wave** block. In order to get $4x$, we grab the output of the **Integrator** (x) and boost it by changing the Gain value to "4" Then, using the **Sum** component, these terms are added, or subtracted, and fed into the integrator. The **Scope** is used to plot the output of the **Integrator** block, $x(t)$. That is the main idea behind solving this system using the model in Figure 1.6.

For this example, we implement the following detailed steps in Simulink:

- Drag needed blocks into the model region [Figure 1.7.]:
  - **Integrator** block from the Continuous group;
  - **Sum** block from the Math Operations group,
  - **Gain** block from the Math Operations group,
  - **Sine Wave** block from the Math Operations group; and,
  - **Scope** block from the Sink group.
- Connect the output of the **Sum** block to the input of the **Integrator** block. [Figure 1.8.]
- Connect the **Integrator** to the **Scope** by clicking on the **Integrator** output and dragging to the **Scope** until they are connected. In more recent versions it is easier to double-click the unattached arrow to get a connection.
- Right-click the **Gain** control and choose **Flip Block** under **Rotate & Flip**. Double-click the **Gain** block and change the **Gain** block value from 1 to 4. It should change on the control.

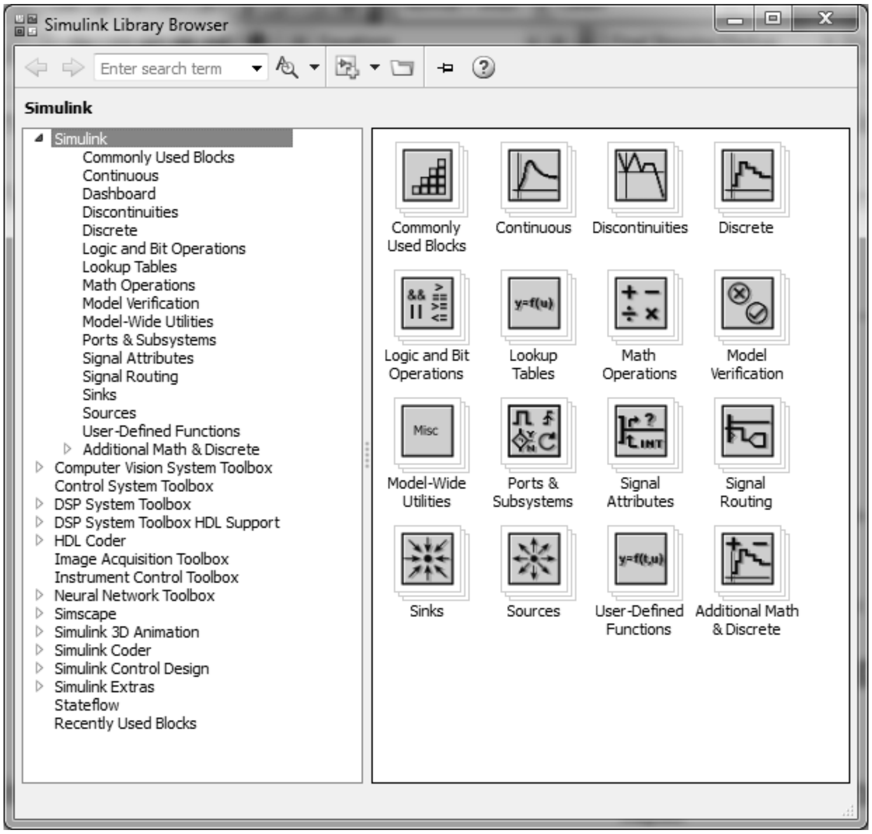


Figure 1.5: The Simulink Library Browser. This is where various blocks can be found for constructing models. [As seen in MATLAB 2015a.]

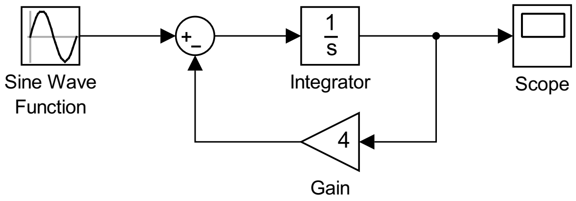


Figure 1.6: System for solving first order ODE $\frac{dx}{dt} = 2\sin 3t - 4x$ as a Simulink simulation.

- Double-click the **Sum** control to bring up **Block Parameters** as shown in Figure 1.9 and change from `|++` to `|+-` in order to set addition/subtraction nodes. [Note that the symbol '`|`' is a blank node. Also, one can change the block to rectangular form. This is often useful in displaying an overall flow direction to the model. In this case the spacer, `|`, is not needed.]

- Double-click the **Sine Wave** block and change the frequency to 3 rad/s and the amplitude to 2. [See Figure 1.10] Set the time dropdown menu to **Use Simulation Time**.

- Connect the **Gain** output to the negative input of **Sum** and the **Sine Wave** output to the positive input on the Sum control. [Note: The Gain can be set to a negative value and connected to a + node in the **Sum** block to obtain the same effect.]

- To add a node to route an x value to the **Gain**, hold the **CTRL key** and click on the Output line of the **Integrator** and drag towards the input

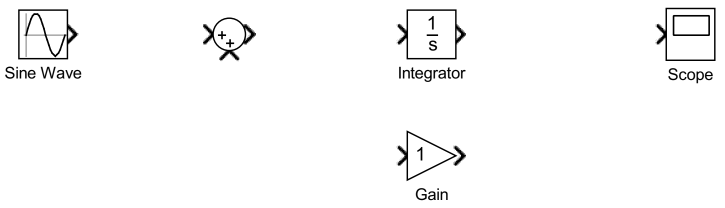


Figure 1.7: Add needed components to the model window.

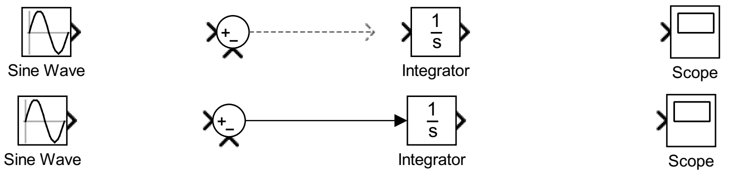


Figure 1.8: Example of connecting two components: Align the components, Click on output of one and drag to another. Then, release to finalize connection. Sometimes it is easier to double-click the temporary arrow to connect the blocks.

of the **Gain**. You can also Right-Click the line where you want the node and drag from there to the **Gain** block. See Figure 1.11.

- The initial value, $x(0)$, of x is inserted by double-clicking the **Integrator** and setting the value. For this example we set $x(0) = 0$.

- One can annotate the diagram by clicking near where labels are needed and typing in the text box. This leads to the model in Figure 1.12. In more current versions the default is to hide block names.

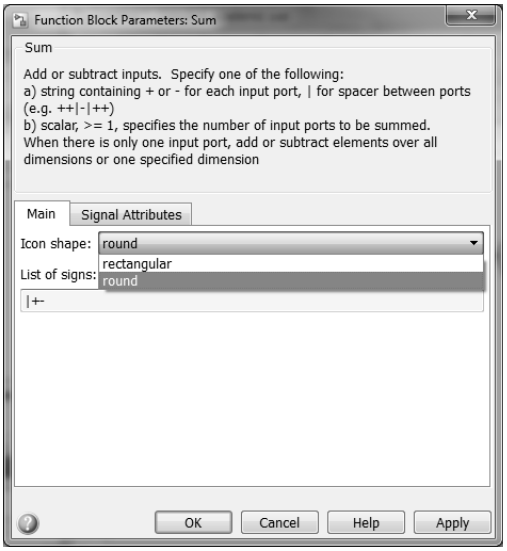
- Save the file under a useable file name. This file can be called in MATLAB, or one can use the run button to run the simulation.

- Double-click the **Scope** to see the solution. Figure 1.13 shows the **Scope** plot after using the autoscale (⛶) feature to rescale the scope view. A little effort is needed to change the plot attributes and to import the plots into working documents. This will be discussed in Section 1.4.

- Also, one can make further changes to the system by checking the **Configuration Parameters** under the Simulation menu item. See Figures 1.14-1.15. In particular, changing the **Refine Factor** can lead to smoother solutions. The solution shown in Figure 1.13 had a setting of 1 and that in Figure 1.16 is the result of setting the **Refine Factor** to 10.

In Release 2017b the names of blocks are hidden. One can change this by going to the block Properties and changing the block parameter for 'HideAutomaticName' or change the model parameter 'HideAutomaticNames'.

As noted in setting the initial value, one can double-click the **Integrator** block and set the initial condition. However, sometimes it is useful to externally feed the initial condition into the block. Double-click the **Integrator** block and change the initial condition source from internal to external. This adds another input to the block. Drag a **Constant** block from the Sources group into the model, connect it to the new input, and change the constant value to the desired initial value. This results in the simulation shown in Figure 1.17.



Function Block Parameters: Sum

Sum

Add or subtract inputs. Specify one of the following:
a) string containing + or - for each input port, | for spacer between ports (e.g. ++|-|++)
b) scalar, >= 1, specifies the number of input ports to be summed.
When there is only one input port, add or subtract elements over all dimensions or one specified dimension

Icon shape: round

List of signs: |+-

Figure 1.9: Block Parameters for the **Sum** control. In many cases it is best to also select the rectangular shape over the default round shape.

1.2 Handling Time in First Order Differential Equations

In this section we review the solutions of first order differential equations, separable first order differential equations and linear first order differential equations involving explicit time dependence. The time dependent functions are obtained using the **Clock** block and a **Math Function** block. Double-clicking the **Math Function** block allows for the selection of a number of common functions.

The independent variable is obtained using the **Clock** block.

Example 1.1. Solve the initial value problem

$$\frac{dy}{dt} = \frac{2}{t}y, \text{ where } y(1) = 1. \tag{1.2}$$

This is a separable equation. Placing y-variables on the left and t-variables on the right side, we have

$$\int \frac{dy}{y} = \int \frac{2}{t}\,dt.$$

Integrating both sides,

$$\ln|y| = 2\ln|t| + C = \ln t^2 + C.$$

Exponentiating, we obtain the general solution,

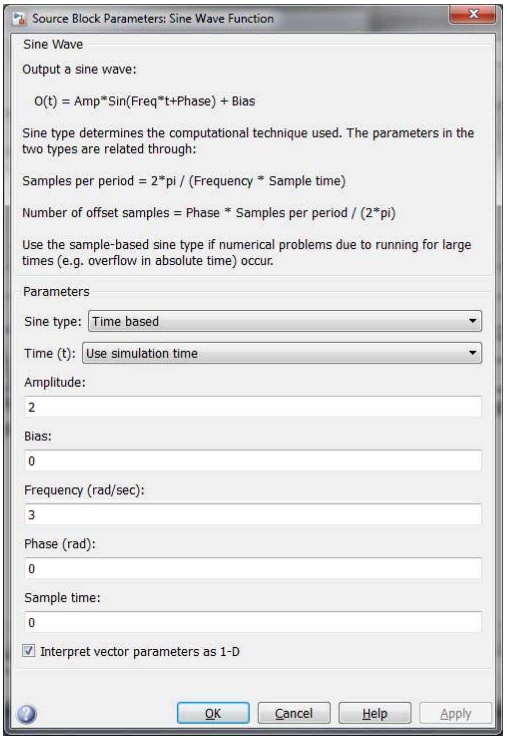
$$y(t) = At^2,$$

where $A = \pm e^C$.

Using the initial condition, we have the solution, $y(t) = t^2$.

We can set up the problem in Simulink as shown in Figure 1.18 for the initial value problem

$$\frac{dy}{dt} = \frac{2}{t}y,$$



Source Block Parameters: Sine Wave Function

Sine Wave

Output a sine wave:

O(t) = Amp*Sin(Freq*t+Phase) + Bias

Sine type determines the computational technique used. The parameters in the two types are related through:

Samples per period = 2*pi / (Frequency * Sample time)

Number of offset samples = Phase * Samples per period / (2*pi)

Use the sample-based sine type if numerical problems due to running for large times (e.g. overflow in absolute time) occur.

Parameters

Sine type: Time based

Time (t): Use simulation time

Amplitude: 2

Bias: 0

Frequency (rad/sec): 3

Phase (rad): 0

Sample time: 0

☑ Interpret vector parameters as 1-D

Figure 1.10: Parameters for the **Sine Wave** block. Select the amplitude and frequency desired.

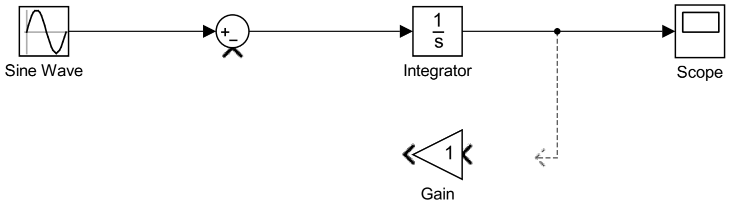


Figure 1.11: Add a node by right-clicking one the line and dragging to the input of a block.

where $y(1) = 1$. Running the simulation, we obtain the solution shown in Figure 1.19.

The solution looks like $y(t) = t^2$. We can verify this by plotting t^2 along with the solution t see if they are the same. Another method would be to compute the difference between the numerical and exact solution, $y(t) - t^2$. In order to do this, we add a **Math Function** block, selecting the square function and connect it to the time route and a **Sum** Block. The solution is also fed into the latter block and the difference is fed into a second **Scope** Block. This is shown in Figure 1.20.

The result of the simulation is shown in Figure 1.21. We note that this is the numerical error, though the solution is only off by 1.4×10^{-5} over the given interval. Considering that the solution at $t = 10$ is $Y(10) = 100$, this is a relative error of roughly 10^{-7}. That seems perfectly acceptable.

It is simple to change the differential equation (1.2) in the previous example to a linear first order differential equation.

$$\frac{dy}{dt} = \frac{2}{t}y + t^2.$$

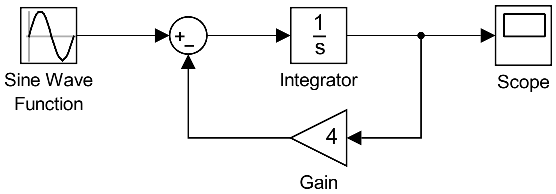


Figure 1.12: Connections for First Order ODE model for $\frac{dx}{dt} = 2\sin 3t - 4x$.

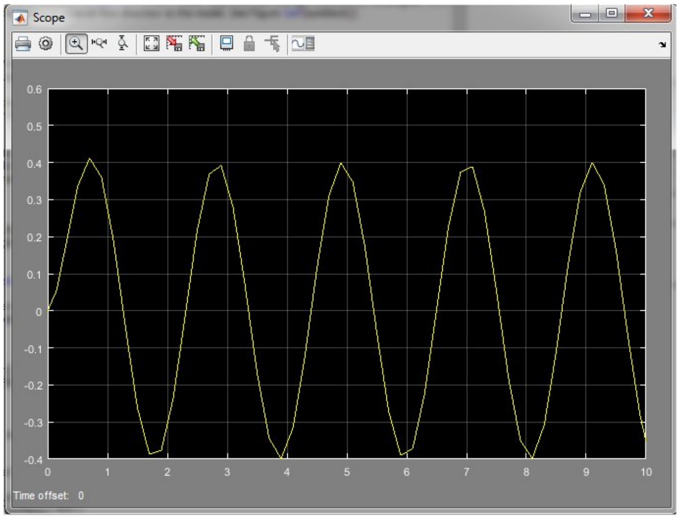


Figure 1.13: **Scope** plot of the solution of $\frac{dx}{dt} = 2\sin 3t - 4x$, $x(0) = 0$, with **Refine Factor**= 1.

Example 1.2. Solve the linear first order differential equation,

$$\frac{dy}{dt} = \frac{2}{t}y + t^2, \tag{1.3}$$

satisfying $y(1) = 1$.

We first rewrite Equation (1.3) in standard form,

$$\frac{dy}{dt} - \frac{2}{t}y = t^2. \tag{1.4}$$

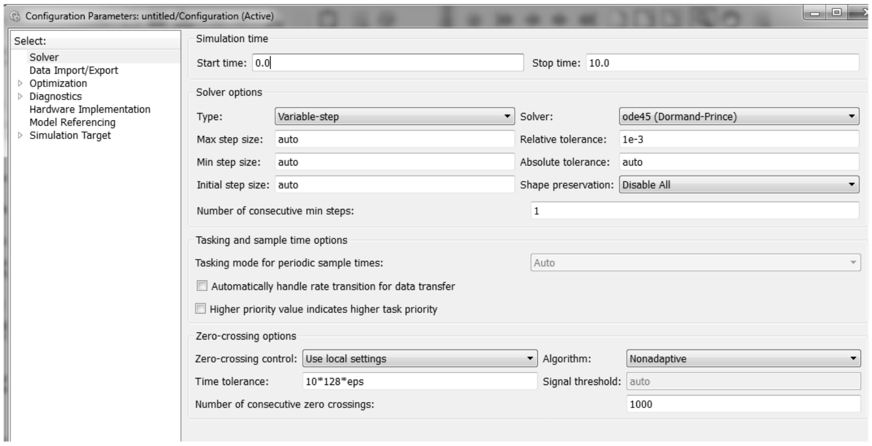
We can now determine the integrating factor,

$$\begin{aligned} \mu(t) &= \exp\left[-\int^t \frac{2}{\tau}\,d\tau\right] \\ &= \exp\left[-2\ln t\right] \\ &= t^{-2}. \end{aligned}$$

Multiplying Equation (1.4) by the integrating factor, $\mu(t)$, we can find the solution:

$$\begin{aligned} t^{-2}\left(\frac{dy}{dt} - \frac{2}{t}y\right) &= t^{-2}t^2 \\ \frac{d}{dt}\left(t^{-2}y\right) &= 1 \\ t^{-2}y(t) &= t + C \\ y(t) &= t^3 + Ct^2. \qquad (1.5) \end{aligned}$$

Using the initial condition, $y(1) = 1$, we obtain $C = 0$. Therefore, the solution is $y(t) = t^3$.



Configuration Parameters: untitled/Configuration (Active)

Simulation time

Start time: 0.0   Stop time: 10.0

Solver options

Type: Variable-step   Solver: ode45 (Dormand-Prince)

Max step size: auto   Relative tolerance: 1e-3

Min step size: auto   Absolute tolerance: auto

Initial step size: auto   Shape preservation: Disable All

Number of consecutive min steps: 1

Tasking and sample time options

Tasking mode for periodic sample times: Auto

☐ Automatically handle rate transition for data transfer

☐ Higher priority value indicates higher task priority

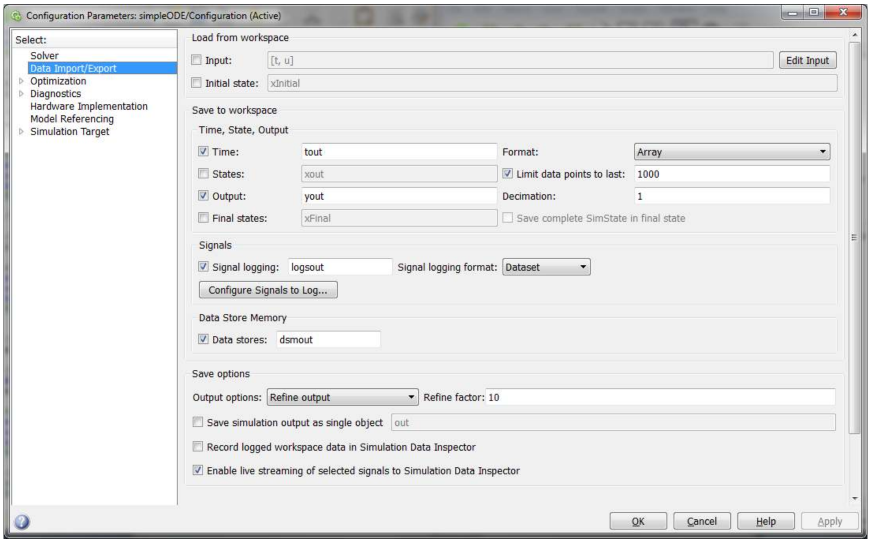
Zero-crossing options

Zero-crossing control: Use local settings   Algorithm: Nonadaptive

Time tolerance: 10*128*eps   Signal threshold: auto

Number of consecutive zero crossings: 1000

Figure 1.14: System Configuration Parameters.



Configuration Parameters: simpleODE/Configuration (Active)

Load from workspace

☐ Input: [t, u]

☐ Initial state: xInitial

Save to workspace

Time, State, Output

☑ Time: tout   Format: Array

☐ States: xout   ☑ Limit data points to last: 1000

☑ Output: yout   Decimation: 1

☐ Final states: xFinal   ☐ Save complete SimState in final state

Signals

☑ Signal logging: logsout   Signal logging format: Dataset

Data Store Memory

☑ Data stores: dsmout

Save options

Output options: Refine output   Refine factor: 10

☐ Save simulation output as single object   out

☐ Record logged workspace data in Simulation Data Inspector

☑ Enable live streaming of selected signals to Simulation Data Inspector

Figure 1.15: Configure Data Import/Export Parameters. Changing the**Refine Factor** can lead to smoother solutions.

The model for this problem is shown in Figure 1.22. Running the simulation, we obtain the numerical solution, $y(t) = t^3$, as shown in Figure 1.23. Computing the difference between the numerical and exact solutions in this case, we find the error is about 6×10^{-5}.

Example 1.3. Consider the initial value problem,

$$\frac{dx}{dt} = 2\sin 3t - 4x, \quad x(0) = 0. \tag{1.6}$$

This is the example that we first solved using Simulink. It is another linear first order differential equation. In standard form is is written as

$$\frac{dx}{dt} + 4x = 2\sin 3t.$$

The integrating factor is found to be

$$\mu(t) = \exp\left[\int 4\,dt\right] = e^{4t}.$$

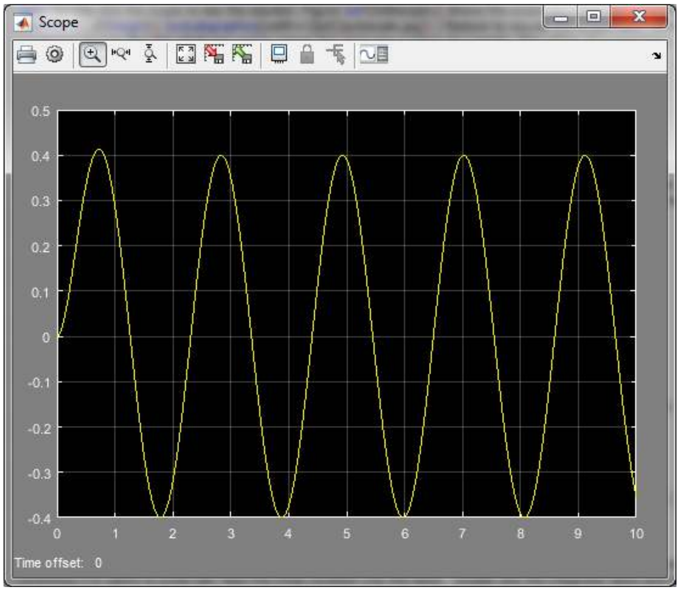


Figure 1.16: **Scope** plot of the solution of $\frac{dx}{dt} = 2\sin 3t - 4x$, $x(0) = 0$, with **Refine Factor**= 10.

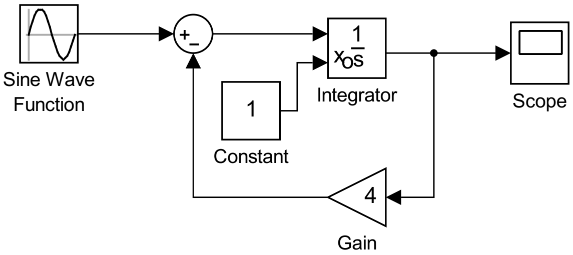


Figure 1.17: Connections for the First Order ODE model for $\frac{dx}{dt} = 2\sin 3t - 4x$ showing how to provide an external initial value.

Multiplying Equation (1.6) by the integrating factor, we can obtain the general solution:

$$\begin{aligned}
\frac{d}{dt}\left(e^{4t}x\right) &= 2e^{4t}\sin 3t \\
e^{4t}x &= 2\int e^{4t}\sin 3t\,dt + C \\
&= \frac{2}{25}e^{4t}\left(4\sin 3t - 3\cos 3t\right) + C \\
x(t) &= \frac{2}{25}\left(4\sin 3t - 3\cos 3t\right) + Ce^{-4t}. \qquad (1.7)
\end{aligned}$$

Using the initial condition, $x(0) = 0$, we find $C = \frac{6}{25}$. Therefore, the particular solution is

$$x(t) = \frac{2}{25}\left(4\sin 3t - 3\cos 3t\right) + \frac{6}{25}e^{-4t}. \qquad (1.8)$$

The solution can be found using Simulink. The model for this exact solution is shown in Figure 1.24. The plot on the scope matches the solution we obtained earlier as seen in Figure 1.13.

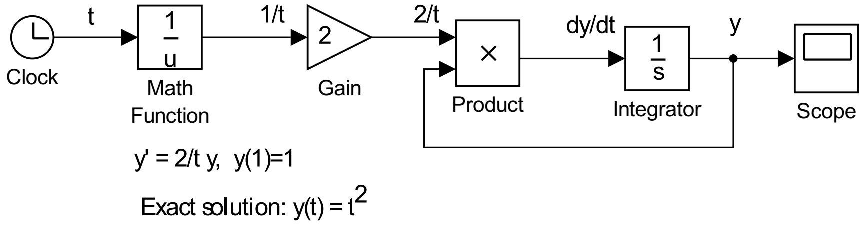


Figure 1.18: First order separable differential equation model.

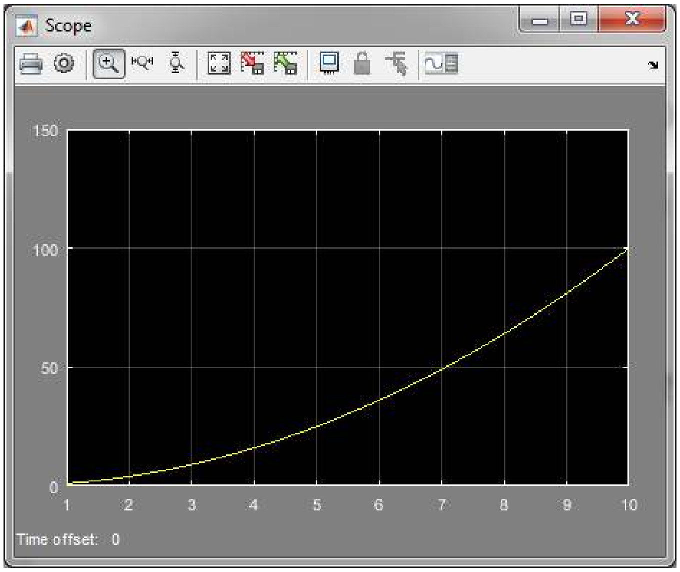


Figure 1.19: **Scope** plot of the solution of initial value problem (1.2), $\frac{dy}{dt} = \frac{2}{t}y$, where $y(1) = 1$.

1.3 Working with Simulink Output

Often we might want to access the solutions in MATLAB. Using the model in Figure 1.18 for a first oder separable equation, we can add the **To Workspace** block. This is shown in Figure 1.25. Double-click and rename the variable as y and change the output type to array. When you run the simulation, it will send the data to MATLAB for further analysis or plotting. This will put **tout** and **y** data into the MATLAB workspace.

In MATLAB you can plot the data using **plot(tout,y)**. You can add labels with **xlabel('t')**, **ylabel('y')**, **title('y vs t')**. Adding the command **set(gcf,'Color',[1,1,1])** makes the plot background white. The result is shown in Figure 1.26.

```
plot(tout,y)
xlabel('t')
ylabel('y')
title('y vs t')
set(gcf,'Color',[1,1,1])
```

Once you have exported your data to the MATLAB workspace and created a plot, then you can use the menu items under **Tools** to annotate the plot. Once you are satisfied with the figure, go to the Edit menu and

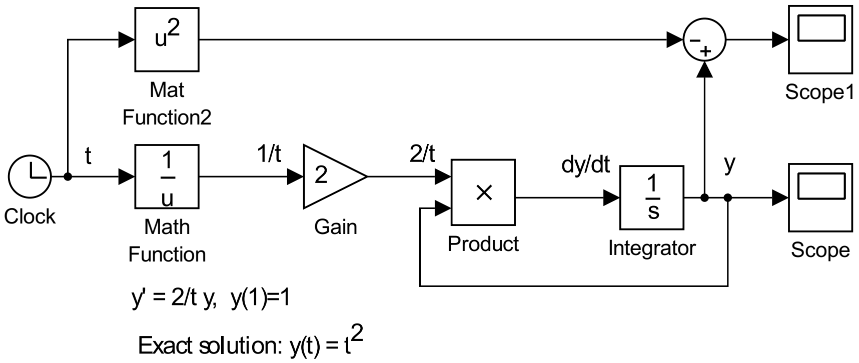


Figure 1.20: First order separable differential equation model with extra blocks to plot the difference between the numerical and exact solution, $y(t) - t^2$, for Equation (1.2).

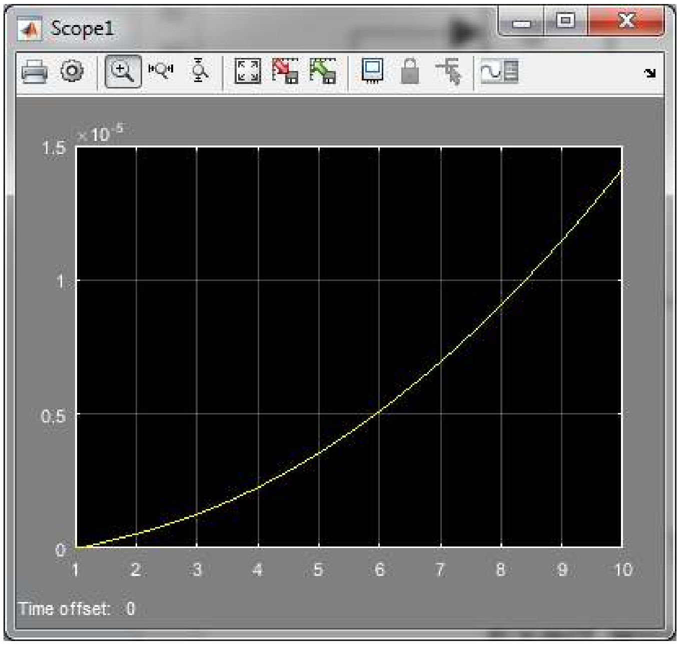


Figure 1.21: **Scope** plot of the difference between the numerical and exact solution, $y(t) - t^2$, for Equation (1.2)

select **Copy Figure**. Go to your report document and **Paste** (CTRL-V) the figure into your document. You can then resize the figure, center it, and add a numbered Figure caption describing the figure. Other methods for recording Simulink Scope images and the Simulink model are described next.

1.4 Printing Simulink Scope Images

In this section we discuss different methods for transferring the plots generated in Simulink models to a document or report. For example, you might want to copy images produced by the scope or your model into an MS Word document. There are several ways you can do this. You might be able to use the Print icon to print to a file or printer, or you can follow one of the following methods. **Note:** In 2015 it was not easy to export plots from Simulink. In the 2017 versions, it is easier to do so and perhaps the

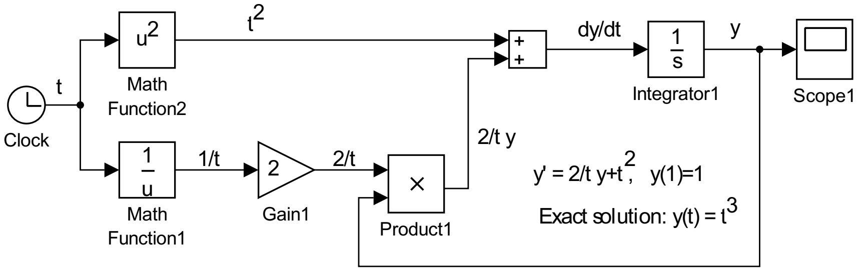


Figure 1.22: Linear first order differential equation model.

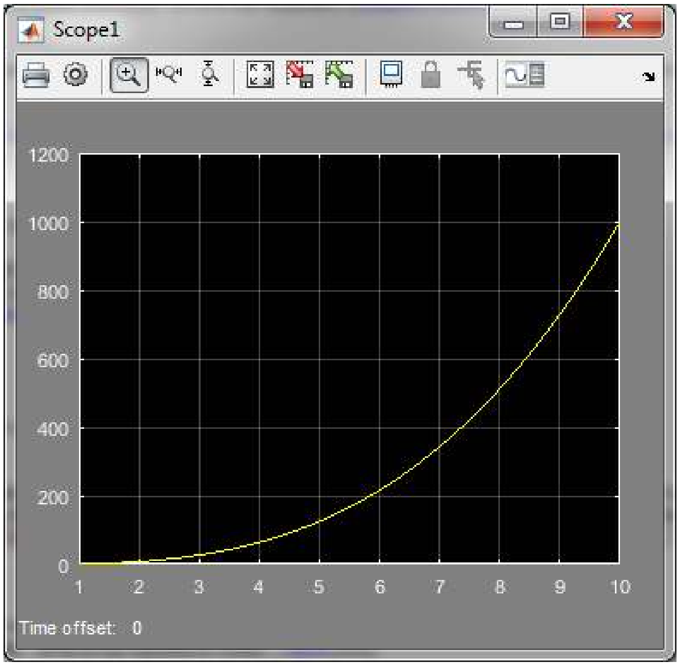


Figure 1.23: **Scope** plot of the difference between the numerical and exact solution, $y(t) - t^2$.

preferred method unless you are using earlier methods.

We compare the 2017a **Scope** figure windows to those shown later from 2015a. In Figure 1.27 one might see slight differences in the Scope icons. In Figure 1.28 the **File** menu shows a menu item for **Print to Figure**. Here one can put the scope figure in a MATLAB figure environment and **Save As** a figure file of different types such as the png image in Figure 1.29. Other methods are provided below for producing output useful for reports.

Method 1:

Select the **Scope** figure window in Figure 1.30, then hit **ALT+PrintScrn** to copy the figure to a clipboard and paste the figure into your application.

Changing the scope appearance.

You might want to change the colors before copying the scope image. Click the **Scope Parameters** icon (2nd icon) and go to the **Style** tab as seen in Figure 1.31. Change the Figure Color to black, Axes Colors to white background and black writing, and Line Color to black. The selection of these parameters is shown in Figure 1.31.

Now the **Scope** plot looks like Figure 1.32.

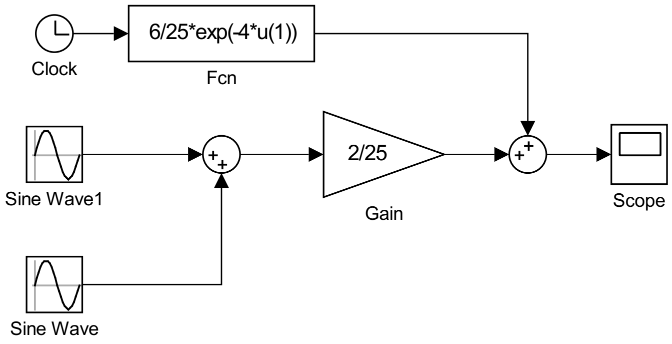


Figure 1.24: Model for plotting the exact solution (1.8) of the initial value problem (1.6).

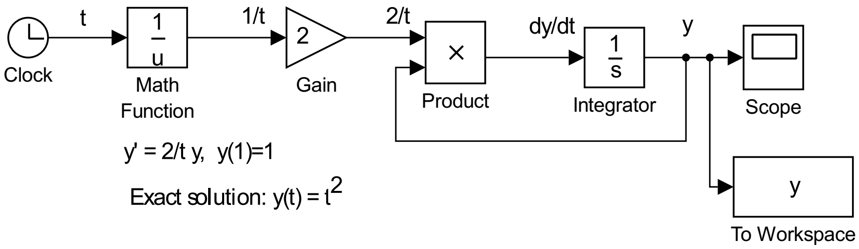


Figure 1.25: Adding To Workspace block for sending output to MATLAB.

Method 2:

Go to **Scope Parameters** and select the **History** tab. Check the **Save data to workspace** box. Note the variable name. Let's change the name to MyScopeData for this example. Saving with **Structure with time** will save the data as a structure. Run the simulation again.

Now, go into the MATLAB command window. You should see the MyScopeData data in the variable list. Type

```
plot(MyScopeData.time, MyScopeData.signals.values)
```

This gives the MATLAB plot in Figure 1.34 which can be manipulated and saved or copied as an image.

Method 3:

You can save the scope image as a jpg image. Create the MATLAB code in Table 1.1. Save this code as an m-file with a name like **prfig.m**. In MATLAB run prfig (type prfig in the Command Window.) It should produce the file 'mypic.jpg in your MATLAB folder. Of course, you can change the name of the image before running **prfig.m** before inserting the figure into your MS Word document as a Picture file.

Method 4:

You can add a **To Workspace** block to your simulation. This will automatically place the data in the MATLAB space. Go to the Simulink library and add a **To Workspace** block to your model as discussed in the last section. Connect this block to the input of the **Scope** (Right-click the input line and drag to connect to the **To Workspace** block.) This will give the connection as shown in Figure 1.36.

Add **To Workspace** block.

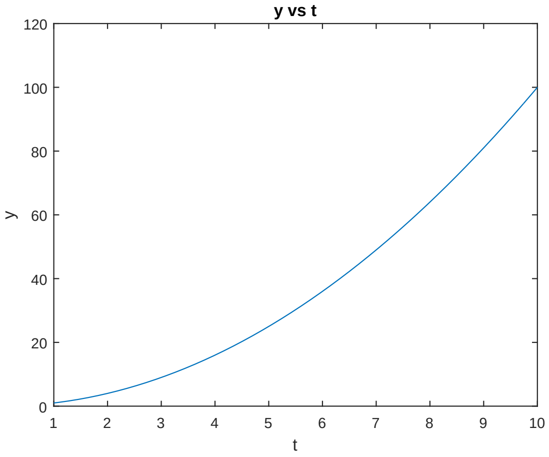


Figure 1.26: Plot of model solution in MATLAB.

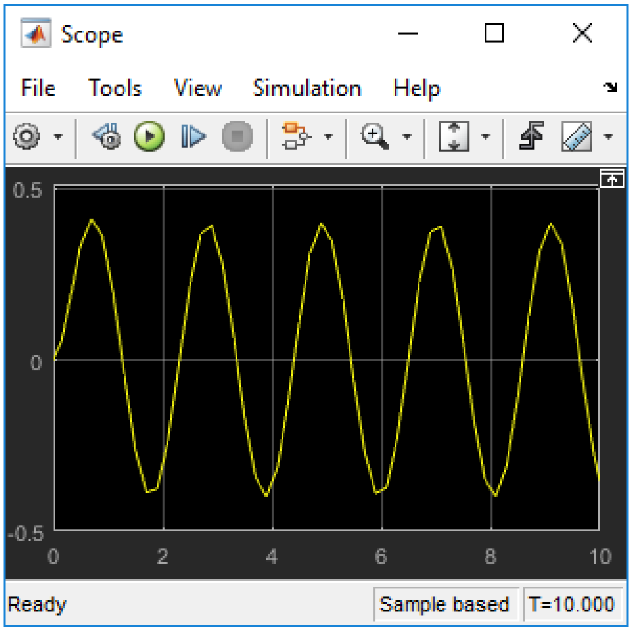


Figure 1.27: **Scope** plot using MATLAB 2017a. Note the differences in some icons.

You can double-click this block and change the variable name that will be saved. Let's assume it is simout. Then, run the simulation. Go into MATLAB and type

```
plot(simout.time,simout.data)
```

This will give you a plot of the **Scope** data. Now you can print, save as an image, or copy (under Edit) to an MS Word document. Below is what you get using Copy Figure under the Edit menu item in the Figure window.

Printing Models

Once you have made a model, you might want to include it in a report. It is easy to capture a model, but a complicated model might not print large enough to see the component annotations.

Printing models.

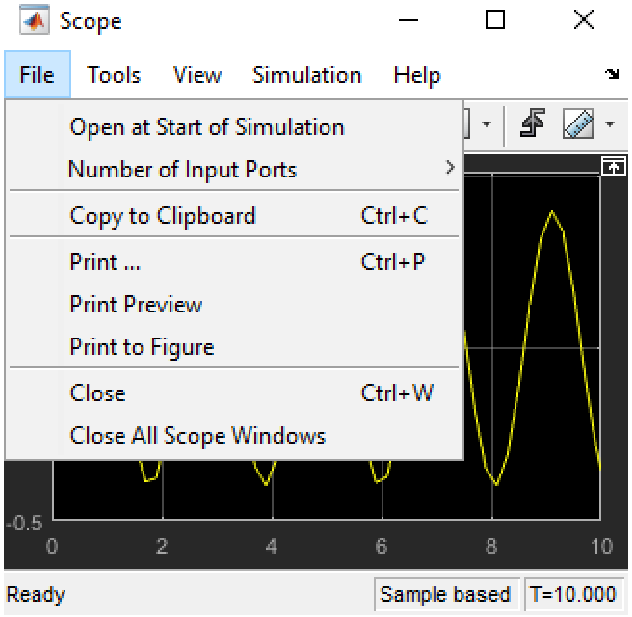


Figure 1.28: **Scope** plot showing File menu where one can Print to Figure.

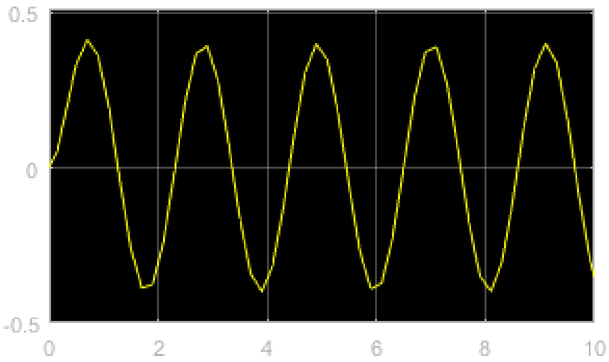


Figure 1.29: MATLAB png image saved as png file. The colors and other attributes can be changed before saving the **Scope** plot.

First open the desired model. Then, in MATLAB you can use the **print** command to print the model. For example, typing the following in the MATLAB command window prints the open model to an encapsulated postscript file:

```
print -s -deps -r300 mymodel.eps
```

For jpg files, you can use

```
print -s -djpeg -r300 mymodel.jpg
```

For other formats, consult the MATLAB help system.

1.5 Scilab and Xcos

THERE ARE ALTERNATIVES TO USING MATLAB. One example is Xcos. Xcos is part of Scilab. Scilab is free and open source software for numerical

Xcos is part of Scilab, an open source alternative.

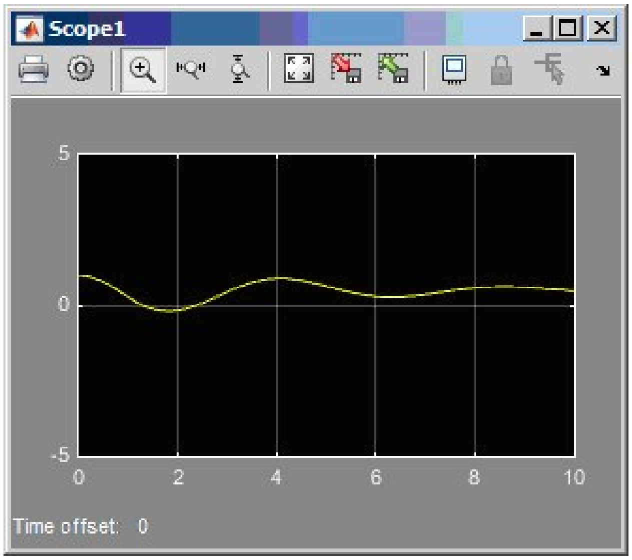


Figure 1.30: **Scope** plot. Note that the plots in this section are generated by the oscillator model in the next chapter.

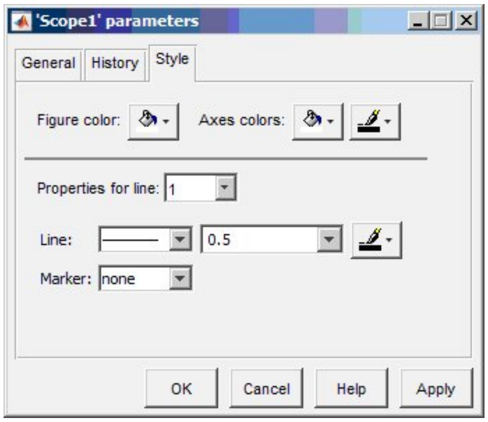


Figure color:

Axes colors:

Properties for line: 1

Line: 0.5

Marker: none

Figure 1.31: **Scope** color parameters.

computation similar to MATLAB. Xcos is a graphical design environment. The Xcos environment is shown in Figure 1.38.

After downloading and installing Scilab from `http://www.scilab.org/`, one can type **xcos** or click on the icon to launch Xcos. This brings up the Xcos Palettes browser and Xcos workspace as shown in Figures 1.39 and 1.40. This looks similar to Simulink's Library Browser as shown in Figure 1.5.

In Figure 1.41 we show the model for solving the first example of this chapter:

$$\frac{dx}{dt} = 2\sin 3t - 4x, \quad x(0) = 0.$$

This is equivalent to the Simulink model in Figure 1.6. We see that this model is similar to the Simulink construction. However, there are are some differences. First of all, the block have a different appearance.

Next, there are some differences in setting up the block parameters. The **Sum** block is set up by double-clicking the block and entering the signs and number of input ports as [1;-1]. This indicates that the **Sum** block has

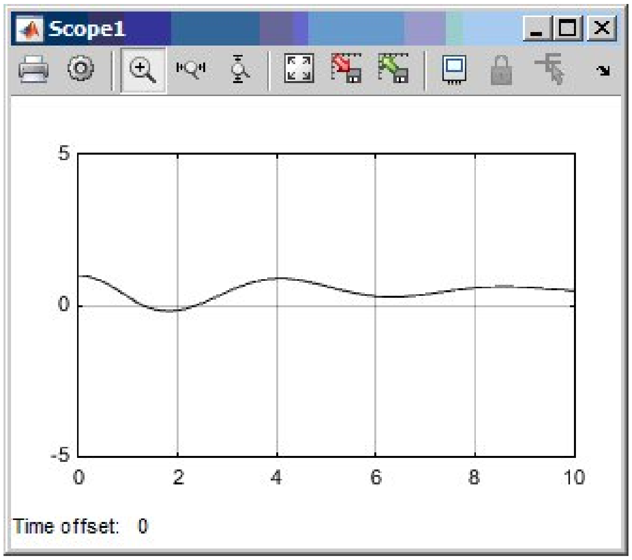
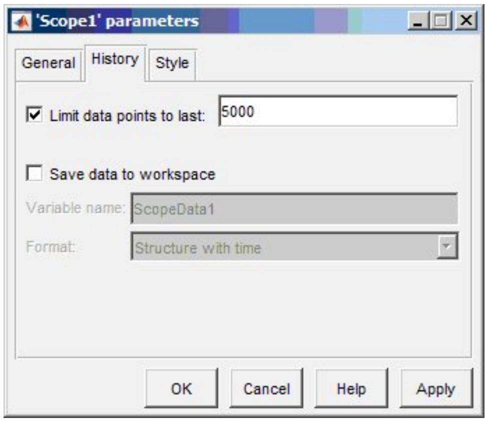


Figure 1.32: **Scope** plot with a white background.



☑ Limit data points to last: 5000

□ Save data to workspace

Variable name: ScopeData1

Format: Structure with time

Figure 1.33: History tab in **Scope parameters**.

two inputs. The first is positive and the second is negative.

The scope requires an additional input. Namely the time is entered using a clock. In Simulink this is automatic, though we had also used the clock to introduce time as an independent variable when needed.

The initial condition and the sine function parameters are entered by double-clicking the integrator and sine block, respectively.

In order to run the simulation, one can click the "play" icon or select **Start** under the **Simulation** menu item. The ODE solver can be changed through **Setup** under the **Simulation** menu. The solution is shown in Figure 1.42.

The blocks are not labeled like Simulink. One can label the blocks by right-clicking and selecting **Edit** under the **Format** item. There one can enter text to appear with the block. Annotation of the workspace is done by selecting a **Text_f** block and adding text to it and changing the fontsize. Sample annotations are shown in Figure 1.43.[4]

We spent time earlier discussing how to capture images of the output and models for reports. In Xcos it is a simple matter to Export the

[4] In newer Simulink versions the block labels disappear. This can be changed by going into the block Properties and changing the model parameters 'HideAutomaticName' or 'ShowName.'

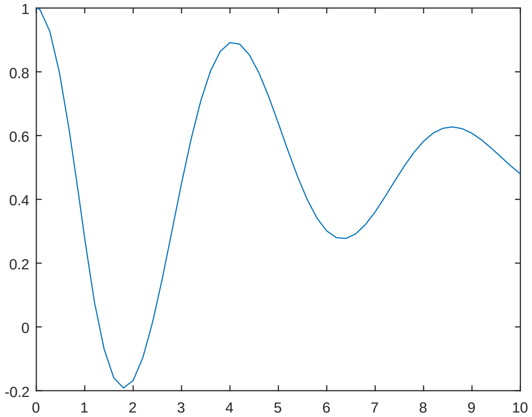


Figure 1.34: Plot generated by Method 2.

Table 1.1: MATLAB code for saving the scope image as a jpg image.

```
shh = get(0,’ShowHiddenHandles’);
set(0,’ShowHiddenHandles’,’On’)
set(gcf,’PaperPositionMode’,’auto’)
set(gcf,’InvertHardcopy’,’off’)
saveas(gcf,’mypic.jpg’)
set(0,’ShowHiddenHandles’,shh)
```

model or the solutions by selecting **Export** under the **File** menu. There are options for saving these to different formats. The images can also be modified by changing the axis range, fonts, colors, etc.

1.6 First Order ODEs in MATLAB

One can use MATLAB to obtain solutions and plots of solutions of differential equations. This can be done either symbolically, using **dsolve**, or numerically, using numerical solvers like **ode45**. In this section we will provide examples of using these to solve first order differential equations. We will end with the code for drawing direction fields, which are useful for looking at the general behavior of solutions of first order equations without explicitly finding the solutions.

Symbolic Solutions

The function **dsolve** obtains the symbolic solution and **ezplot** is used to quickly plot the symbolic solution. As an example, we apply **dsolve** to solve the main model in this chapter.

At the MATLAB prompt, type the following:

```
 sol = dsolve(’Dx=2*sin(t)-4*x’,’x(0)=0’,’t’);
```

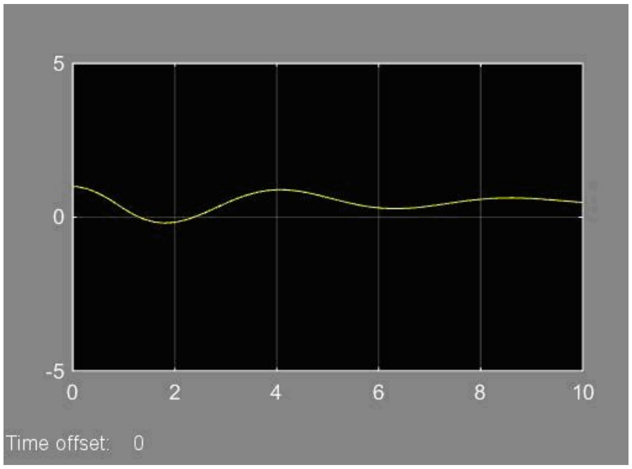


Figure 1.35: Scope plot from Method 3.

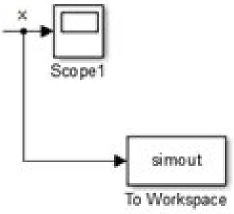


Figure 1.36: Use of a **To Workspace** block.

```
ezplot(sol,[0 10])
xlabel('t'),ylabel('x'), grid
```

The solution is given as

```
 sol =


  (2*exp(-4*t))/17 - (2*17^(1/2)*cos(t + atan(4)))/17
```

Figure 1.44 shows the solution plot.

Another approach to symbolically solving a differential equation is provided in the following code snippet. The dsolve command symbolically solves the equation given by 'eq' and using the initial condition in 'cond.' It then plots the solution. This results in the same plot and solution obtained earlier in the chapter.

```
syms t y(t)
dy = diff(y,t);
eq = dy + 4*y == 2*sin(3*t);
cond = y(0) == 0;
sol(t) = dsolve(eq,cond)
fplot(sol,[0 10])
xlabel('t')
ylabel('y')
title('dy/dt-4y = t')
```

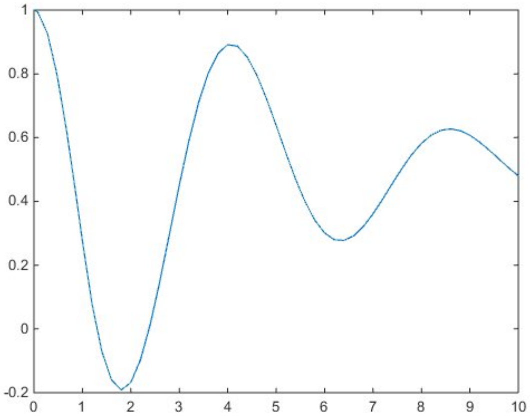
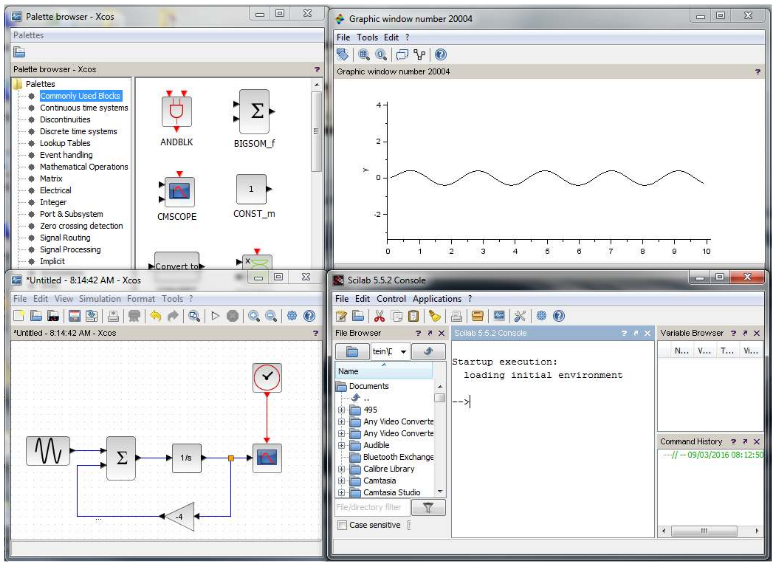


Figure 1.37: **Scope** plot from Method 4.



```
Startup execution:
  loading initial environment

-->
```

Figure 1.38: The Xcos workspace.

ODE45 and Other Solvers.

There are several ODE solvers in MATLAB, implementing Runge-Kutta and other numerical schemes. Examples of its use are in the differential equations textbook. For example, one can implement **ode45** to solve the initial value problem

$$\frac{dy}{dt} = -\frac{yt}{\sqrt{2-y^2}}, \quad y(0) = 1,$$

using the following code:

```
[t y]=ode45('func',[0 5],1);
plot(t,y)
xlabel('t'),ylabel('y')
title('y(t) vs t')
```

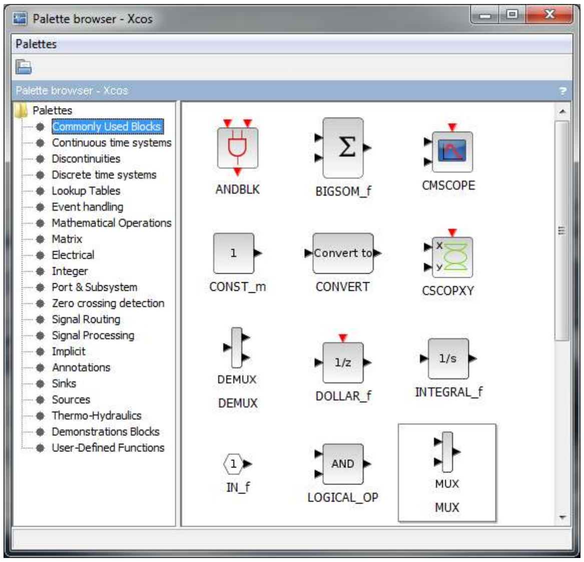


Figure 1.39: The Xcos Palette browser.

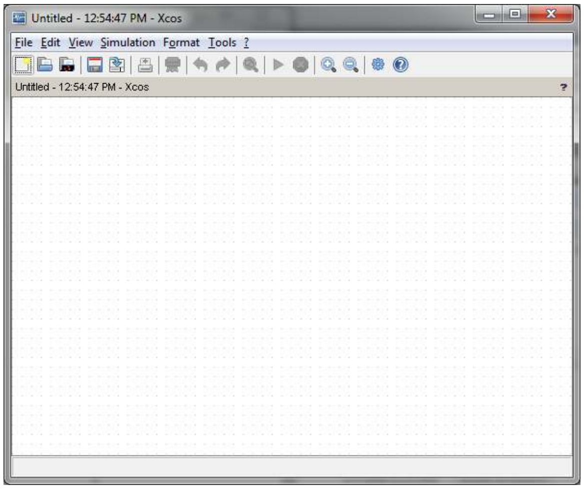

Untitled - 12:54:47 PM - Xcos
File Edit View Simulation Format Tools ?
Untitled - 12:54:47 PM - Xcos

Figure 1.40: The Xcos workspace.

One can define the function **func** in a file **func.m** such as

```
function f=func(t,y)
f=-t*y/sqrt(2-y.^2);
```

Running the above **ode45** code produces Figure 1.45.
One can also use **ode45** to solve higher order differential equations. Second order differential equations are discussed in Chapter 3 Section 5. See MATLAB help for other examples and other ODE solvers.

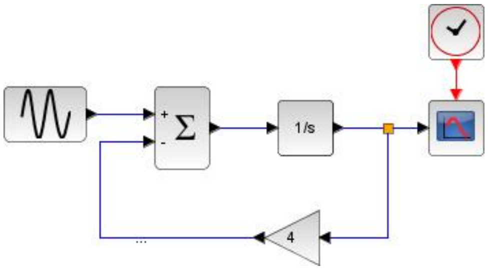


Figure 1.41: The Xcos model for solving the first order ODE $\frac{dx}{dt} = 2\sin 3t - 4x$.

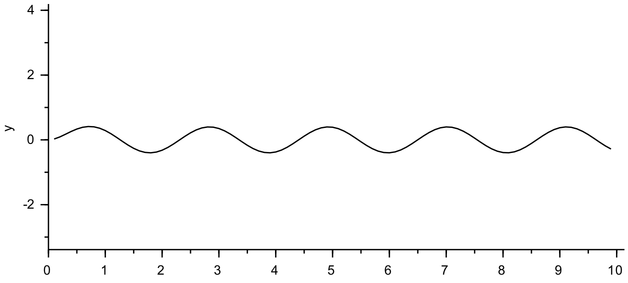


Figure 1.42: The Xcos model solution of $\frac{dx}{dt} = 2\sin 3t - 4x$, $x(0) = 0$.

Direction Fields

ONE CAN PRODUCE DIRECTION FIELDS in MATLAB. For the differential equation

$$\frac{dy}{dx} = f(x,y),$$

we note that $f(x,y)$ is the slope of the solution curve passing through the point in the xy=plane. Thus, the direction field is a collection of tangent vectors at points (x,y) indication the slope, $f(x,y)$, at that point.

A sample code for drawing direction fields in MATLAB is given by

```
[x,y]=meshgrid(0:.1:2,0:.1:1.5);
dy=1-y;
dx=ones(size(dy));
quiver(x,y,dx,dy)
axis([0,2,0,1.5])
xlabel('x')
ylabel('y')
```

The mesh command sets up the xy-grid. In this case x is in $[0,2]$ and y is in $[0,1.5]$. In each case the grid spacing is 0.1.

We let dy = 1-y and dx =1. Thus,

$$\frac{dy}{dx} = \frac{1-y}{1} = 1-y.$$

The **quiver** command produces a vector (dx,dy) at (x,y). The slope of each vector is dy/dx. The other commands label the axes and provides a window with xmin=0, xmax=2, ymin=0, ymax=1.5. The result of using the above code is shown in Figure 1.46.

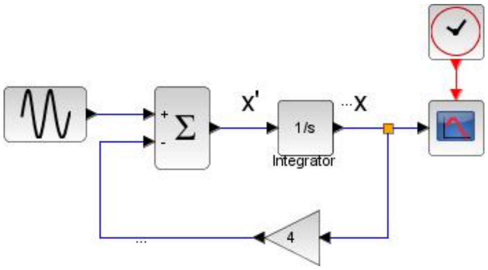


Figure 1.43: The Xcos model with annotation added.

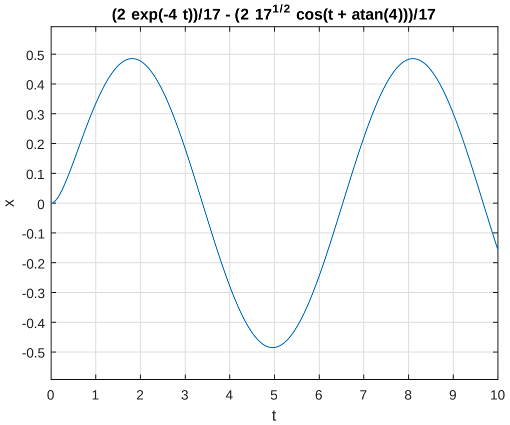


Figure 1.44: The solution of Equation (1.1) with $x(0) = 0$ found using MATLAB's **dsolve** command.

One can add solution, or integral, curves to the direction field for different initial conditions to further aid in seeing the connection between direction fields and integral curves. One needs to add to the direction field code the following lines:

```
hold on
[t,y] = ode45(@(t,y) 1-y, [0 2], .5);
plot(t,y,'k','LineWidth',2)
[t,y] = ode45(@(t,y) 1-y, [0 2], 1.5);
plot(t,y,'k','LineWidth',2)
hold off
```

Here the function $f(t,y) = 1 - y$ is entered this time using MATLAB's anonymous function, **@(t,y) 1-y**. Before plotting, the **hold** command is invoked to allow plotting several plots on the same figure. The result is shown in Figure 1.47

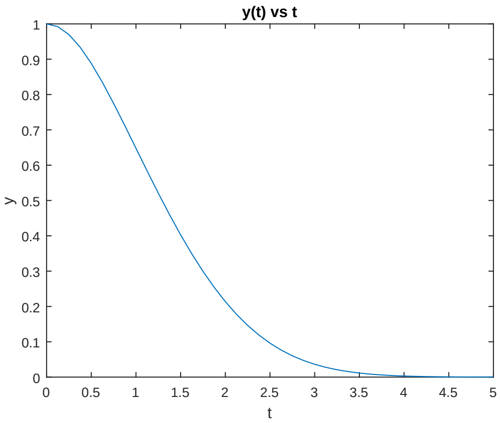


Figure 1.45: A plot of the solution of $\frac{dy}{dt} = -\frac{yt}{\sqrt{2-y^2}}, \quad y(0) = 1$, found using MATLAB's **ode45** command.

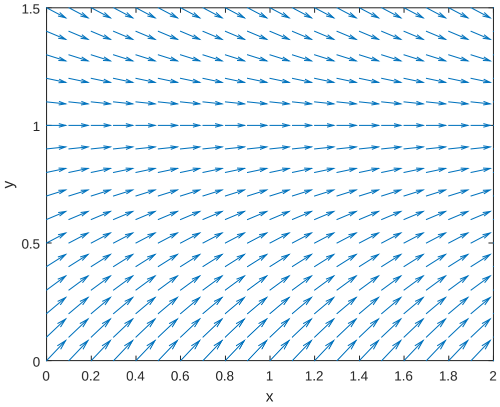


Figure 1.46: A direction field produced using MATLAB's **quiver** function for $y' = 1 - y$.

1.7 Exercises

1. Construct the model in Figure 1.6 for solving the initial value problem $\frac{dx}{dt} = 2\sin 3t - 4x$, $x(0) = 0$, and produce a plot of the solution.

2. Modify the model in the Problem 1. to solve $\dfrac{dx}{dt} = f(t) - 2x$ for a different function, $f(t)$ and initial condition.

3. Solve the following initial value problems using MATLAB's **dsolve** command (See Section 1.6) and Simulink. Provide plots of the solutions for both cases. How do the solutions compare?

   a. $y' = xy$, $y(0) = 1$.
   b. $y' = 2y(3 - y)$, for different initial conditions, $y(0) = 4$, $y(0) = 2$, and $y(0) = -1$.
   c. $y' = 1 + x + y$, $y(0) = 1$.
   d. $y' = (y^2 - 4)(y - 4)$ for different initial conditions, $y(0) = 5$, $y(0) = 3$, $y(0) = 1$, $y(0) = -1$, and $y(0) = -3$.

4. Use MATLAB to plot direction fields for the following:

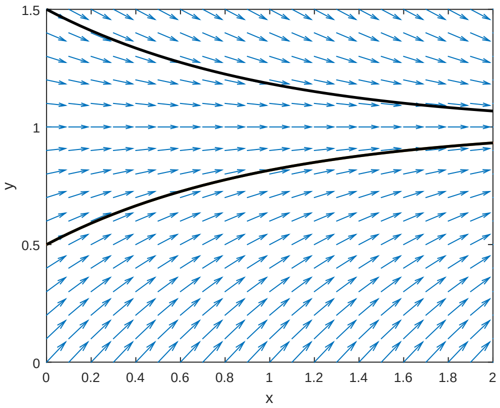


Figure 1.47: A direction field produced using MATLAB's **quiver** function for $y' = 1 - y$ with solution curves added.

a. $y' = xy$.

b. $y' = 2y(3 - y)$.

c. $y' = 1 + x + y$.

d. $y' = (y^2 - 4)(y - 4)$.

5. Solve the following initial value problems using one of MATLAB's numerical ODE solvers like **ode45**. Plot the solutions and compare with the corresponding solutions in Problem 3.

a. $y' = xy,\ y(0) = 1$.

b. $y' = 2y(3 - y)$, for different initial conditions, $y(0) = 4$, $y(0) = 2$, and $y(0) = -1$.

c. $y' = 1 + x + y,\ y(0) = 1$.

d. $y' = (y^2 - 4)(y - 4)$ for different initial conditions, $y(0) = 5$, $y(0) = 3$, $y(0) = 1$, $y(0) = -1$, and $y(0) = -3$.