2

First Order Differential Equations

We have seen how to solve simple first order differential equations using Simulink. In particular we have solved initial value problems for the equations

$$\frac{dy}{dt} = \frac{2}{t}y, \quad y(1) = 1, \tag{2.1}$$

$$\frac{dy}{dt} = \frac{2}{t}y + t^2, \quad y(1) = 1, \tag{2.2}$$

$$\frac{dx}{dt} = 2\sin 3t - 4x, \quad x(0) = 0. \tag{2.3}$$

The Simulink models were provided in Figures 1.18, 1.22, and 1.6, respectively.

In this chapter we solve a few more first order equations in the form of applications. These will include growth and decay, Newton's Law of Cooling, pursuit curves, free fall and terminal velocity, the logistic equation, and the logistic equation with delay.

2.1 Exponential Growth and Decay

The simplest differential equations are those governing growth and decay. As an example, we will discuss population models.

Let $P(t)$ be the population at time t. We seek an expression for the rate of change of the population, $\frac{dP}{dt}$. Assuming that there is no migration of population, the only way the population can change is by adding or subtracting individuals in the population. The equation would take the form

$$\frac{dP}{dt} = \text{Rate In} - \text{Rate Out}.$$

The *Rate In* could be due to the number of births per unit time and the *Rate Out* by the number of deaths per unit time. The simplest forms for these rates would be given by terms proportional to the population:

$$\text{Rate In } = bP \quad \text{and} \quad \text{Rate Out } = mP.$$

Here we have denoted the birth rate as b and the mortality rate as m. This gives the total rate of change of population as

$$\frac{dP}{dt} = bP - mP \equiv kP, \tag{2.4}$$

where $k = b - m$.

Equation (2.4) is easily modeled in Simulink. All of the needed blocks are under the Commonly Used Blocks group. We need an **Integrator**, **Constant**, **Gain**, and a **Scope** block. The output from the **Integrator** can be feed into a **Gain** control, which represents k, and the output from the **Gain**, kP, can then be used as an input to the **Integrator**. We add the **Scope** in order to plot the solution. The model is shown in Figure 2.1. Note that a **Constant** block was added to provide an external input of the initial condition.

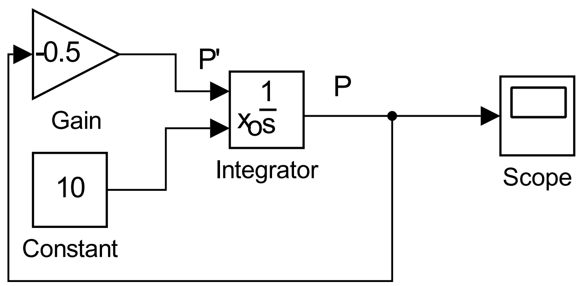

Figure 2.1: Simulink model for exponential growth and decay. The initial value, $P(0) = 10$, is set in the **Constant** block and $k = -0.5$ is set in the **Gain**.

The solution for exponential decay with $P(0) = 10$ and $k = -0.5$ is shown in Figure 2.2. The simulation time was set at 10s.

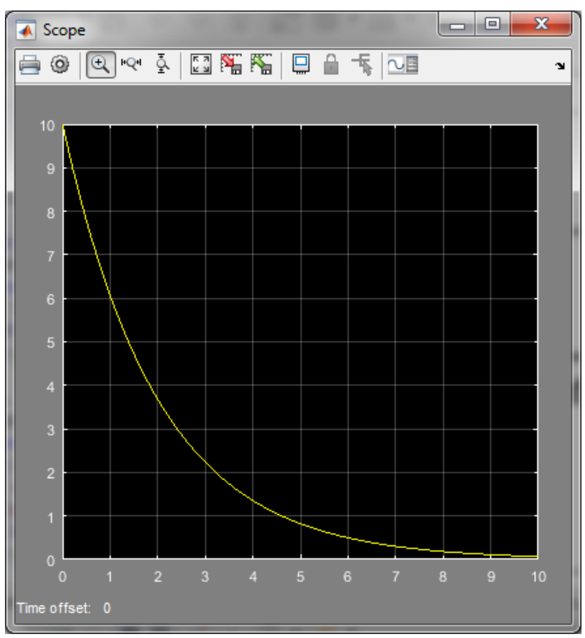

Figure 2.2: Solution for the exponential decay with $P(0) = 10$ and $k = -0.5$. The simulation time was set at 10.

The exact solution is easily found noting that Equation (2.4) is a separable equation. Rearranging the equation, its differential form is

$$\frac{dP}{P} = k\,dt.$$

Integrating, we have

$$\int \frac{dP}{P} = \int k\,dt$$

$$\ln |P| = kt + C. \tag{2.5}$$

Next, we solve for $P(t)$ through exponentiation,

$$\begin{aligned} |P(t)| &= e^{kt+C} \\ P(t) &= \pm e^{kt+C} \\ &= Ae^{kt}. \end{aligned} \tag{2.6}$$

Here we have defined the arbitrary constant, $A = \pm e^C$.

If the population at $t = 0$ is P_0, i.e., $P(0) = P_0$, then the solution gives $P(0) = Ae^0 = A = P_0$. So, the solution of the initial value problem is

$$P(t) = P_0 e^{kt}.$$

In the Simulink model, the initial value was given as $P(0) = 10$ and the decay constant by $k = -0.5$. Therefore, the solution in Figure 2.2 is of the function $P(t) = 10e^{-0.5t}$.

Equation (2.4) is the familiar exponential model of population growth:

Malthusian population growth.

$$\frac{dP}{dt} = kP.$$

We obtained solutions exhibiting exponential growth $(k > 0)$ or decay $(k < 0)$. This Malthusian growth model has been named after Thomas Robert Malthus (1766-1834), a clergyman who used this model to warn of the impending doom of the human race if its reproductive practices continued. Later we modify this model to account for competition for resources, leading to the logistic differential equation.

2.2 Newton's Law of Cooling

IF YOU TAKE YOUR HOT CUP OF TEA, and let it sit in a cold room, the tea will cool off and reach room temperature after a period of time. The law of cooling is attributed to Isaac Newton (1642-1727) who was probably the first to state results on how bodies cool.[1] The main idea is that a body at temperature $T(t)$ is initially at temperature $T(0) = T_0$. It is placed in an environment at an ambient temperature of T_a. The goal is to find the temperature at a later time, $T(t)$.

[1] Newton's 1701 Law of Cooling is an approximation to how bodies cool for small temperature differences $(T - T_a \ll T)$ and does not take into account all of the cooling processes. One account is given by C. T. O'Sullivan, Am. J. Phys (1990) p 956-960.

We will assume that the rate of change of the temperature of the body is proportional to the temperature difference between the body and its surroundings. Thus, we have

$$\frac{dT}{dt} \propto T - T_a.$$

The proportionality is removed by introducing a cooling constant,

$$\frac{dT}{dt} = -k(T - T_a), \tag{2.7}$$

where $k > 0$.

This differential equation can be solved by first rewriting the equations as

$$\frac{d}{dt}(T - T_a) = -k(T - T_a).$$

This now takes the form of exponential decay of the function $T(t) - T_a$. The solution is easily found as

$$T(t) - T_a = (T_0 - T_a)e^{-kt},$$

or

$$T(t) = T_a + (T_0 - T_a)e^{-kt}.$$

Example 2.1. A cup of tea at 90^{o}C cools to 85^{o}C in ten minutes. If the room temperature is 22^{o}C, what is its temperature after 30 minutes?

Using the general solution with $T_0 = 90^o$C,

$$T(t) = 22 + (90 - 22)e^{-k} = 22 + 68e^{-kt},$$

we then find k using the given information, $T(10) = 85^o$C. We have

$$\begin{aligned} 85 &= T(10) \\ &= 22 + 68e^{-10k} \\ 63 &= 68e^{-10k} \\ e^{-10k} &= \frac{63}{68} \approx 0.926 \\ -10k &= \ln 0.926 \\ k &= -\frac{\ln 0.926}{10} \\ &\approx 0.00764 \text{ min}^{-1}. \end{aligned}$$

This gives the solution for this model as

$$T(t) = 22 + 68e^{-0.00764t}.$$

Now we can answer the question. What is $T(30)$?

$$T(30) = 22 + 68e^{-0.00764(30)} = 76^o\text{C}.$$

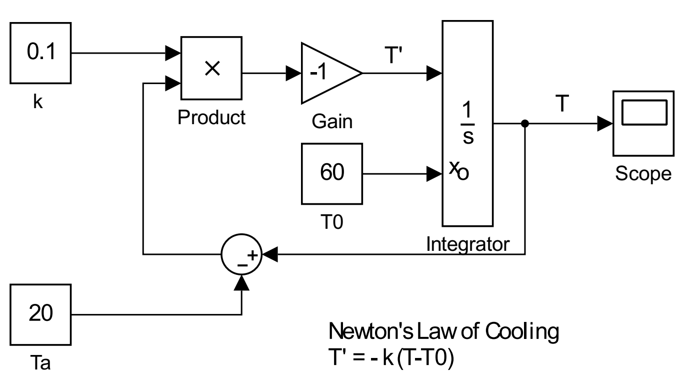

Figure 2.3: Simulation model for Newton's Law of Cooling, $T' = -k(T - T_a)$, $T(0) = T0$. Here we set $k = 0.1$ s^{-1}, $T_a = 20^o$C, and $T_0 = 60^o$C.

Next we model Equation (2.7) in Simulink. The input for the **Integrator** is simply $-k(T - T_a)$. We need to define the constants k and T_a. We will externally input the initial condition, $T(0) = T_0$ in the **Integrator** block. The simple model is shown in Figure 2.3. In this case we set $k = 0.1\ \text{s}^{-1}$, $T_a = 20^o$C, and $T_0 = 60^o$C. Running the simulation for 100 s, we obtain the solution shown in Figure 2.4.

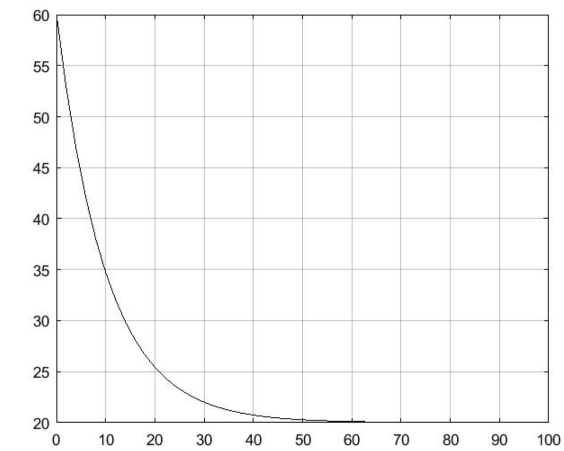

Figure 2.4: Solution of Newton's Law of Cooling example.

How good is the solution? We can solve the problem by hand for this set of parameters. However, we will take this opportunity to introduce the idea of a subsystem and set up a model in which we can interactively modify the constants and get Simulink to automatically provide the exact solution for comparison.

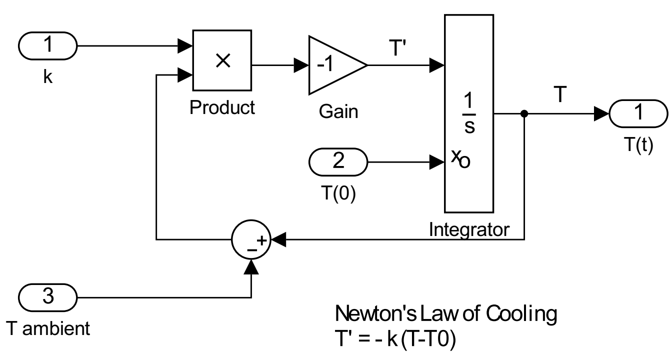

Figure 2.5: Creating a subsystem for the Newton's Law of Cooling model.

We begin by replacing the scope with an output block. The **Out1** block can be found in the Sink group. The input to the subsystem will be the three parameters, k, T_0, and T_a. Each of these constant blocks in Figure 2.3 will be replaced by an **In1** block, found in the Sources group. In Figure 2.5 the three inputs and one output are now oval blocks.

Creating a subsystem.

Double-click each of the three input blocks, one at a time, and set the Port Number of k, T_0, and T_a, to 1, 2, and 3, respectively. Finally, rename each of these controls using the labels that make sense, such as k for k. In

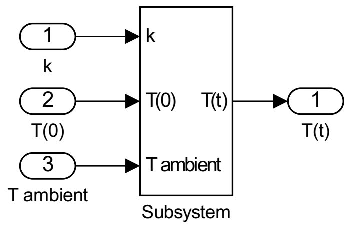

Figure 2.6: Subsystem for Newton's Law of Cooling, $T' = -k(T - T_a)$, $T(0) = T0$.

Figure 2.5 we show the subsystem that we have created.

Now highlight the entire subsystem using **CTRL-A**. In the menu system, look for **Create Subsystem from Selection**. This is under the menu item **Diagram** and subitem **Subsystem & Model Reference**. Rearranging the resulting subsystem, one has something like the subsystem block in Figure 2.6. This is the equivalent of a black box with three inputs and one output.

Next, we can make use of the subsystem just created. Replace the three input ports with constant blocks. Rename the **Constant** blocks with the parameter name and fill each block with a value. The output port can be replaced with a **Scope** block, or any other form of output desired. This can be seen in Figure 2.7.

Before finishing with this model, we will build in the exact solution. Recall that the general solution can be written in terms of the parameters as

$$T(t) = T_a + (T_0 - T_a)e^{-kt}.$$

So, we can feed the values of the parameters in the model into a **Fcn** block and output the exact solution for comparison. We will also need a time value. So, we will need the **Clock** block as well.

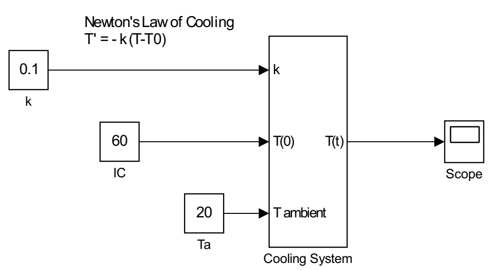

Figure 2.7: Using a user-created subsystem for Newton's Law of Cooling.

The entire model is shown in Figure 2.8. The subsystem is labeled **Cooling System** The top portion is a repetition of the Newton's Law of Cooling model implemented previously.

We have added a **Fcn** block from the User-Defined Functions group. The input will be a vector containing all of the variables in the exact solution. This is accomplished by adding a **Mux** (or Multiplex) block. Double-click the **Mux** block and set the number of inputs to 4.

Now, double-click the **Fcn** block and enter the exact solution in the form

```
u(1)+u(2)*exp(-u(3)*u(4))
```

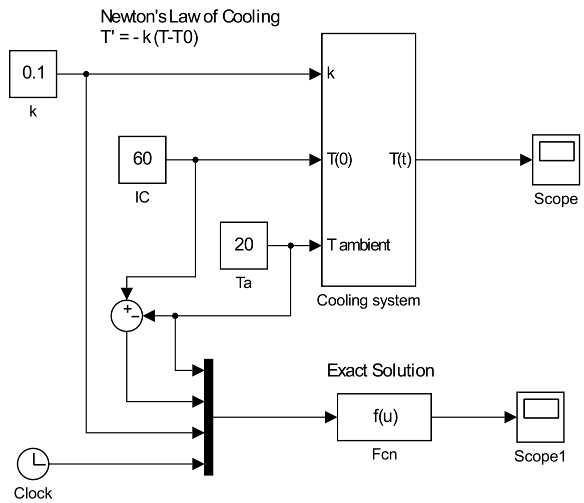

Figure 2.8: Model of Newton's Law of Cooling, $T' = -k(T - T_a)$, $T(0) = T0$, using the subsystem feature.

Here we have assumed that the variables are fed into the **Mux** block in the order T_a, $T_0 - T_a$, k, and t. In Figure 2.8 one can see how the values are routed into the **Mux** block.

The output can be attached to a second scope, as shown, or can be subtracted from the output of the **Cooling System** block to show the closeness of the two solutions. One can also send the output to MATLAB using the **To Workspace** block.

2.3 Free Fall with Drag

CONSIDER AN OBJECT FALLING TO THE GROUND with air resistance? Free fall is the vertical motion of an object solely under the force of gravity. It has been experimentally determined that an object near the surface of the Earth falls at a constant acceleration in the absence of other forces, such as air resistance. This constant acceleration is denoted by $-g$, where g is called the acceleration due to gravity. The negative sign is an indication that we have chosen a coordinate system in which "up" is positive.

We are interested in determining the position, $y(t)$, of a falling body as a function of time. The differential equation governing free fall is have

$$\ddot{y}(t) = -g. \tag{2.8}$$

Note that we will occasionally use a dot to indicate time differentiation.

We need to model the air resistance. As an object falls faster and faster, the resistive force becomes greater. This drag force is a function of the velocity. The idea is to write Newton's Second Law of Motion $F = ma$ in the form

$$m\ddot{y} = -mg + f(v), \tag{2.9}$$

where $f(v)$ gives the resistive force and mg is the weight. Note that this applies to free fall near the Earth's surface. Also, for $f(v)$ to be a resistive force, $f(v)$ should oppose the motion. If the body is falling, then $f(v)$ should be positive. If the body is rising, then $f(v)$ would have to be negative to indicate the opposition to the motion.

We will model the drag as quadratic in the speed, $f(v) = bv^2$.

Example 2.2. Solve the free fall problem with $f(v) = bv^2$.

The differential equation that we need to solve is

$$\dot{v} = kv^2 - g, \tag{2.10}$$

where $k = b/m$. Note that this is a first order equation for $v(t)$.

Formally, we can separate the variables and integrate over time to obtain

$$t + C = \int^v \frac{dz}{kz^2 - g}. \tag{2.11}$$

If we can do the integral, then we have a solution for v. We evaluate this integral using Partial Fraction Decomposition.

In order to factor the denominator in the current problem, we first have to rewrite the constants. We let $\alpha^2 = g/k$ and write the integrand as

$$\frac{1}{kz^2 - g} = \frac{1}{k}\frac{1}{z^2 - \alpha^2}. \tag{2.12}$$

Noting that

$$\frac{1}{kz^2 - g} = \frac{1}{2\alpha k}\left[\frac{1}{z-\alpha} - \frac{1}{z+\alpha}\right], \tag{2.13}$$

the integrand can be easily integrated to find

$$t + C = \frac{1}{2\alpha k}\ln\left|\frac{v-\alpha}{v+\alpha}\right|. \tag{2.14}$$

Solving for v, we have

$$v(t) = \frac{1 - Ae^{2\alpha k t}}{1 + Ae^{2\alpha k t}}\alpha, \tag{2.15}$$

where $A \equiv e^C$. A can be determined using the initial velocity by inserting $t = 0$,

$$v(0) = \frac{1-A}{1+A}\alpha.$$

Then,

$$A = \frac{\alpha - v_0}{\alpha + v_0}.$$

There are other forms for the solution in terms of a tanh function, which the reader can determine as an exercise. One important conclusion is that for large times, the ratio in the solution approaches -1. Thus, $v \to -\alpha = -\sqrt{\frac{g}{k}}$ as $t \to \infty$. This means that the falling object will reach a constant terminal velocity.

Equation (2.10) can be modeled in Simulink. The model is shown in Figure 2.9. The solution for $k = 0.00159\text{m}^{-1}$, which is found for the above sample computation, is shown in Figure 2.10. We see that terminal velocity is obtained and matches the predicted value, $-\sqrt{\frac{g}{k}} = -78$ m/s.

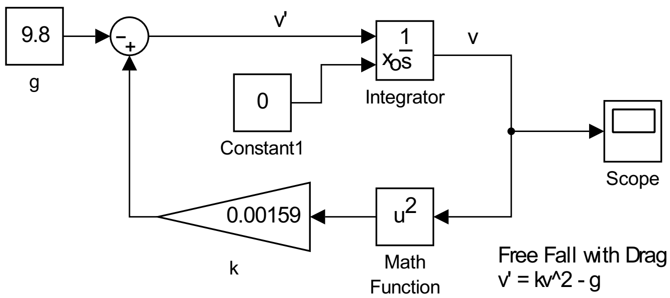

Figure 2.9: Model for free fall with drag as described by $\dot{v} = kv^2 - g$.

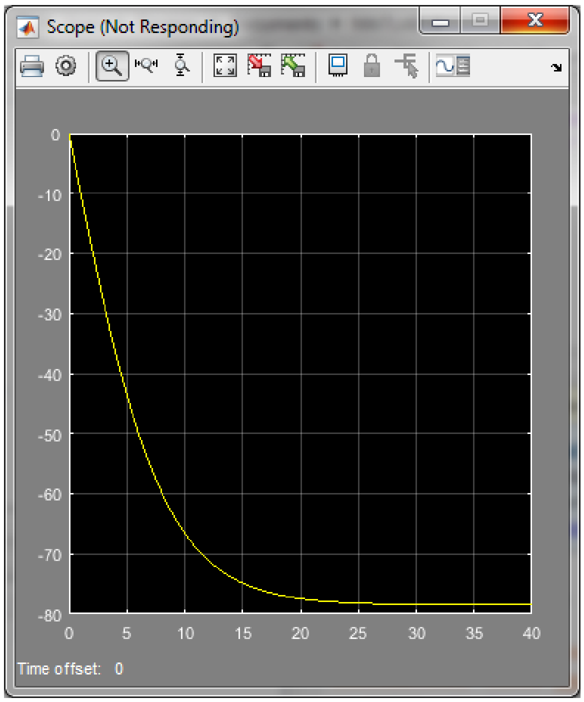

Figure 2.10: Solution for free fall with drag with $k = 0.00159$ starting from rest.

2.4 Pursuit Curves

Another application that is interesting is to find the path that a body traces out as it moves towards a fixed point or another moving body. Such curves are know as pursuit curves. These could model aircraft or submarines following targets, or predators following prey. For example, a hawk follows a sparrow, a large fish chases a small fish, or a fox chases a rabbit.

Example 2.3. A dog at point (x, y) sees a cat traveling at speed v along a straight line. The dog runs towards the cat at constant speed w but always in a direction along line of sight between their positions. If the dog starts out at the point $(0, 0)$ at $t = 0$, when the cat is at $(a, 0)$, then what is the path the dog needs to follow? Will the dog catch the cat?

We show the path in Figure 2.12. Let the cat's path be along the

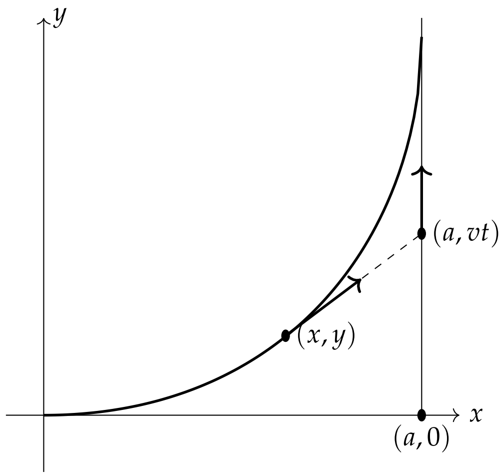

Figure 2.11: A dog at point (x, y) sees a cat at point (a, vt) and always follows the straight line between these points.

line $x = a$. Therefore, the cat is at position (a, vt) at time t. The goal is to find the dog's path, $(x(t), y(t))$, or $y = y(x)$.

First we consider the equation of the line of sight between the points (x, y) and (a, vt). Considering that the slope of this line is the same as the slope of the tangent to the path, $y = y(x)$, we have

$$y' = \frac{vt - y}{a - x}.$$

The dog is moving at a constant speed, w and the distance the dog to travels s given by $L = wt$, where t is the running time from the origin. The distance the dog travels is also given by the arclength of the path between $(0, 0)$ and (x, y) :

$$L = \int_0^x \sqrt{1 + [y'(x)]^2}\, dx.$$

Eliminating the time using $y' = \frac{vt-y}{a-x}$, we have

$$\int_0^x \sqrt{1 + [y'(x)]^2}\, dx = \frac{w}{v}(y + (a - x)y').$$

Furthermore, we can differentiate this result with respect to x to get rid of the integral,

$$\sqrt{1 + [y'(x)]^2} = \frac{w}{v}(a - x)y''. \tag{2.16}$$

This is the differential equation governing the dog's pursuit. A Simulink model of this problem is shown in Figure 2.12.

The full solution for the path is given by

$$y(x) = \frac{a}{2}\left[\frac{\left(\frac{x}{a}\right)^{1+\frac{v}{w}}}{1 + \frac{v}{w}} - \frac{\left(\frac{x}{a}\right)^{1-\frac{v}{w}}}{1 - \frac{v}{w}}\right] + \frac{avw}{w^2 - v^2}.$$

Can the dog catch the cat? This would happen if there is a time when $y(0) = vt$. Inserting $x = 0$ into the solution, we have $y(0) = \frac{avw}{w^2 - v^2} = vt$. This is possible if $w > v$.

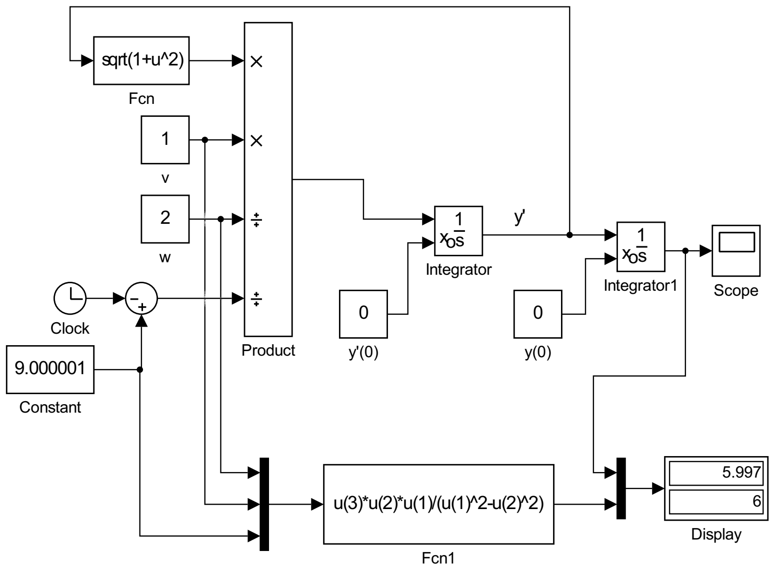

Figure 2.12: Model for the pursuit curve, $(a-x)y'' = \frac{v}{w}\sqrt{1+[y'(x)]^2}$, $y(0) = 0$, $y'(0) = 0$, for $w = 2$ and $v = 1$.

Analytic Solution

For the interested reader, we complete the solution of the problem by noting that Equation (2.16) can be rewritten as a first order separable equation in the slope function $z(x) = y'(x)$. Namely,

$$\frac{w}{v}(a-x)z' = \frac{v}{x}\sqrt{1+z^2}.$$

Separating variables, we find

$$\frac{w}{v}\int \frac{dz}{\sqrt{1+z^2}} = \ln(z+\sqrt{1+z^2})\int \frac{dx}{a-x}.$$

The integrals can be computed using standard methods from calculus.

We can easily integrate the right hand side,

$$\int \frac{dx}{a-x} = -\ln|a-x| + c_1.$$

The left hand side takes a little extra work,[2] or looking the integral to find

$$\int \frac{dz}{\sqrt{1+z^2}} = \ln(z+\sqrt{1+z^2}) + c_2.$$

[2] One can use trigonometric substitution. Let $z = \tan\theta$ and $dz = \sec^2\theta\, d\theta$. Then, the method proceeds as follows:

$$\begin{aligned} \int \frac{dz}{\sqrt{1+z^2}} &= \int \frac{\sec^2\theta}{\sqrt{1+\tan^2\theta}}\, d\theta \\ &= \int \sec\theta\, d\theta \\ &= \ln(\tan\theta + \sec\theta) + c_2 \\ &= \ln(z+\sqrt{1+z^2}) + c_2. \end{aligned}$$

Putting these results together, we have for $x > 0$,

$$\ln(z+\sqrt{1+z^2}) = \frac{v}{w}\ln x + C.$$

Using the initial condition $z = y' = 0$ and $x = a$ at $t = 0$,

$$0 = \frac{v}{w}\ln a + C,$$

or $C = -\frac{v}{w}\ln a$.

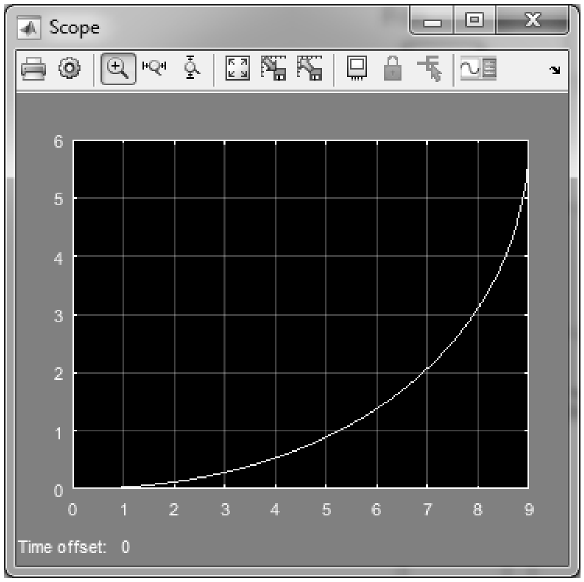

Figure 2.13: Solution for the pursuit curve.

Using this value for c, we find

$$\begin{aligned}\ln(z+\sqrt{1+z^2}) &= \frac{v}{w}\ln x - \frac{v}{w}\ln a \\ &= \ln\left(\frac{x}{a}\right)^{\frac{v}{w}} \\ z+\sqrt{1+z^2} &= \left(\frac{x}{a}\right)^{\frac{v}{w}}. \end{aligned} \tag{2.17}$$

We can solve for $z = y'$, to find

$$y' = \frac{1}{2}\left[\left(\frac{x}{a}\right)^{\frac{v}{w}} - \left(\frac{x}{a}\right)^{-\frac{v}{w}}\right]$$

Integrating,

$$y(x) = \frac{a}{2}\left[\frac{\left(\frac{x}{a}\right)^{1+\frac{v}{w}}}{1+\frac{v}{w}} - \frac{\left(\frac{x}{a}\right)^{1-\frac{v}{w}}}{1-\frac{v}{w}}\right] + k.$$

Since $y(a) = 0$, we can solve for the integration constant, k,

$$k = \frac{a}{2}\left[\frac{1}{1-\frac{v}{w}} - \frac{1}{1+\frac{v}{w}}\right] = \frac{avw}{w^2-v^2}.$$

2.5 The Logistic Equation

In this section we will explore a nonlinear population model. Typically, we want to model the growth of a given population, $y(t)$, and the differential equation governing the growth behavior of this population is developed in a manner similar to that done in the section on growth and decay. Recall the simple population model from Section 2.1,

$$\frac{dy}{dt} = by - my, \tag{2.18}$$

where we had defined the birth rate as b and the mortality rate as m. If these rates are constant, then we can define $k = b - m$ and obtain the familiar exponential model of population growth.

When more realistic populations get large enough, there is competition for resources, such as space and food, which can lead to a higher mortality rate. Thus, the mortality rate may be a function of the population size, $m = m(y)$. The simplest model would be a linear dependence, $m = \tilde{m} + cy$. Then, the previous exponential model would take the form

The logistic model was first published in 1838 by Pierre François Verhulst (1804-1849) in the form

$$\frac{dN}{dt} = rN\left(1 - \frac{N}{K}\right),$$

where N is the population at time t, r is the growth rate, and K is what is called the carrying capacity. Note that in this model $r = k = Kc$.

$$\frac{dy}{dt} = ky - cy^2, \tag{2.19}$$

where $k = b - \tilde{m}$. This is known as the *logistic model* of population growth. Typically, c is small and the added nonlinear term does not kick in until the population gets large enough.

Example 2.4. Show that Equation (2.19) can be written in the form

$$z' = kz(1 - z)$$

which has only one parameter.

We carry this out by rescaling the population, $y(t) = \alpha z(t)$, where α is to be determined. Inserting this transformation, we have

$$\begin{aligned} y' &= ky - cy^2 \\ \alpha z' &= \alpha kz - c\alpha^2 z^2, \end{aligned}$$

or

$$z' = kz\left(1 - \alpha\frac{c}{k}z\right).$$

Thus, we obtain the result, $z' = kz(1 - z)$, if we pick $\alpha = \frac{k}{c}$.

The point of this derivation is to show that there is only one free parameter, k, and that many combinations of c and k in the original problem lead to essentially the same solution up to rescaling.

We can model the logistic equation, $y' = ry(1 - y)$, with $r = 1$ and $y(0) = 0.1$ in Simulink. The model is shown in Figure 2.14. Running the model gives the solution in Figure 2.15. It shows the typical sigmoidal curve bounded by the solutions $y = 0$ and $y = 1$.

2.6 The Logistic Equation with Delay

SOMETIMES THE RATE OF CHANGE does not immediately take place when the system changes. This can be modeled using differential-delay equations. For example, when the resources are being depleted, the effects might be delayed. So, a possible model would be the logistic equation with delay,

$$y' = ry(t)(1 - y(t - \tau)),$$

where τ is a fixed delay time.

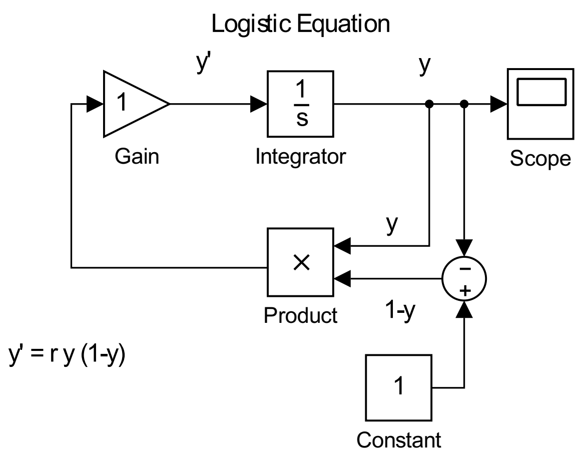

Figure 2.14: Simulink model for the logistic equation, $y' = ry(1-y)$.

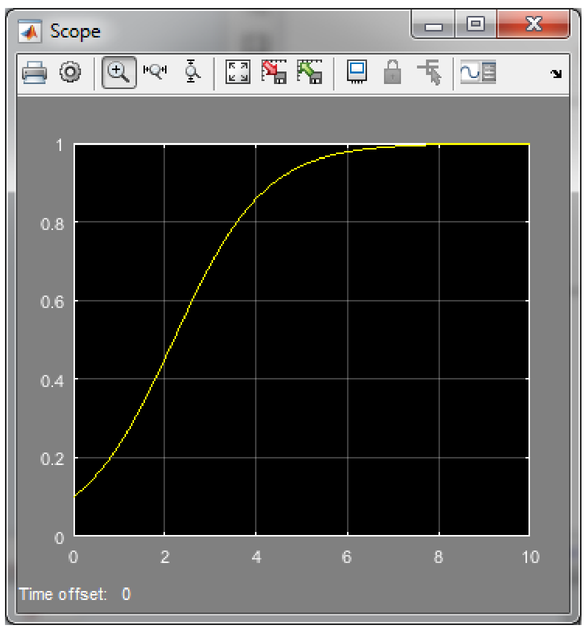

Figure 2.15: Solution of the logistic equation, $y' = ry(1-y)$, with $r = 1$ and $y(0) = 0.1$.

The problem with trying to solve this model at time t is that we need to know something about the solution for earlier times, $y(t-\tau)$. One way to tackle the problem is to specify the solution for times $[0, \tau]$ and then to solve the equation with delay using this starting value. So, if $y = 2$ initially, we could let $y = 2$ for $[0, \tau]$.

The Simulink model is shown in Figure 2.16. A Switch block is used to specify the starting values for times up to $\tau = 1$. Then, the differential equation solver takes over with a Delay block used to enter the delay term. This model produces the solution in Figure 2.17.

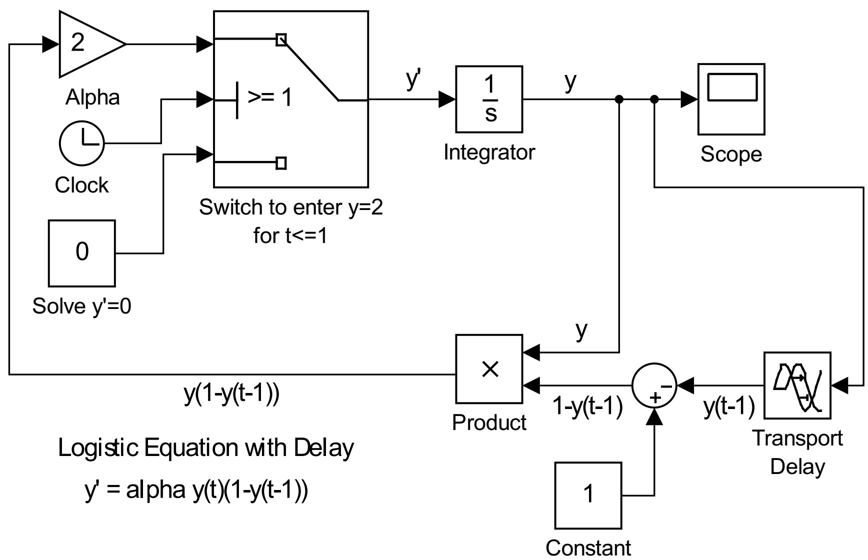

Figure 2.16: Model for the logistic equation with delay, $y' = ry(t)(1 - y(t-\tau))$.

2.7 Exercises

1. Model the following first order differential equations in Simulink and find the solutions for different initial conditions.

   a. $\dfrac{dy}{dx} = \dfrac{e^x}{2y}$.
   b. $\dfrac{dy}{dt} = y^2(1+t^2)$.
   c. $\dfrac{dy}{dx} = \dfrac{\sqrt{1-y^2}}{x}$.
   d. $xy' = y(1-2y)$.
   e. $y' - (\sin x)y = \sin x$.
   f. $xy' - 2y = x^2$.
   g. $\dfrac{ds}{dt} + 2s = st^2$.
   h. $x' - 2x = te^{2t}$.
   i. $\dfrac{dy}{dx} + y = \sin x,$.
   j. $\dfrac{dy}{dx} - \dfrac{3}{x}y = x^3$.

2. Consider the case of free fall with a damping force proportional to the velocity, $f_D = \pm kv$ with $k = 0.1$ kg/s.

   a. Using the correct sign, consider a 50 kg mass falling from rest at a height of 100m. Find the velocity as a function of time. Does the mass reach terminal velocity?
   b. Let the mass be thrown upward from the ground with an initial speed of 50 m/s. Find the velocity as a function of time as it travels upward and then falls to the ground. How high does the mass get? What is its speed when it returns to the ground?

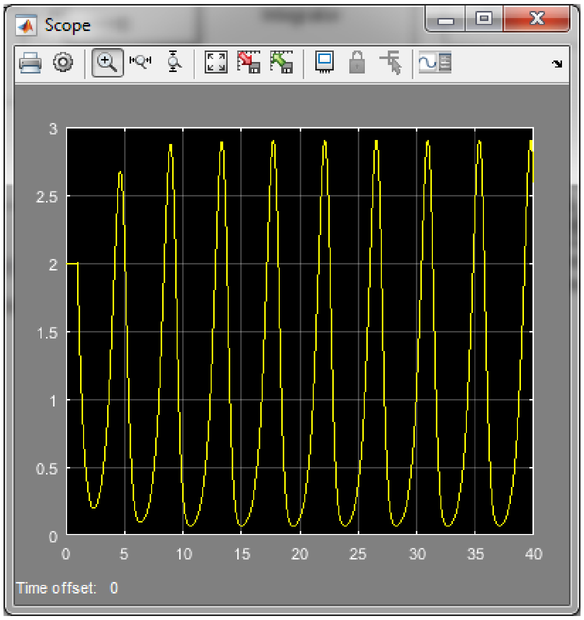

Figure 2.17: Solution of the logistic equation with delay, $y' = ry(t)(1 - y(t-1))$ for $y = 2$, $t \in [0, 1]$.

3. A paratrooper, 322 lbs including munitions, jumps from 10,000 ft. Model this free fall with air resistance $f(v) = 15v^2$ in Simulink. First, write down the free fall equation. Use the model to solve for $v(t)$. Is there a terminal velocity? Find the time to land and the impact velocity.

4. Model the following problem in Simulink: The temperature inside your house is 70^oF and it is 30^oF outside. At 1:00 A.M. the furnace breaks down. At 3:00 A.M. the temperature in the house has dropped to 50^oF. Assuming the outside temperature is constant and that Newton's Law of Cooling applies, determine when the temperature inside your house reaches 40^oF.

5. Model the following problem in Simulink: A body is discovered during a murder investigation at 8:00 P.M. and the temperature of the body is 70^oF. Two hours later the body temperature has dropped to 60^oF in a room that is at 50^oF. Assuming that Newton's Law of Cooling applies and the body temperature of the person was 98.6^oF at the time of death, determine when the murder occurred.