

Differential Equations Review III

I. Laplace Transforms $Y(s) \equiv L\{y(t)\} \equiv \int_0^{\infty} y(t)e^{-st} dt$. Know basic transforms!

a. Properties

$$L\{af(t) + bg(t)\} = aF(s) + bG(s).$$

$$L\left\{\frac{dy}{dt}\right\} = sY(s) - y(0).$$

$$L\left\{\frac{d^2y}{dt^2}\right\} = s^2Y(s) - sy(0) - y'(0).$$

$$L\{e^{at}y(t)\} = Y(s - a).$$

$$L\{H(t - a)y(t - a)\} = e^{-as}Y(s).$$

$$L\{tf(t)\} = -\frac{d}{ds}F(s).$$

b. Transform Pairs

$f(t)$	$F(s)$	$f(t)$	$F(s)$
c	$\frac{c}{s}$	e^{at}	$\frac{1}{s - a}, s > a.$
t^n	$\frac{n!}{s^{n+1}}, s > 0.$	$\sinh at$	$\frac{a}{s^2 - a^2}$
$\sin \omega t$	$\frac{\omega}{s^2 + \omega^2}$	$\cosh at$	$\frac{s}{s^2 - a^2}$
$\cos \omega t$	$\frac{s}{s^2 + \omega^2}$	$\delta(t - a)$	$e^{-as}, a \geq 0, s > 0.$
$H(t - a)$	$\frac{e^{-as}}{s}, s > 0$		

c. Convolution Theorem

i. $(f * g)(t) = \int_0^t f(t - u)g(u) du.$

ii. $L\{f * g\} = F(s)G(s).$

d. Related topics

- i. Partial fraction decomposition
- ii. Completing the Square
- iii. Solving Differential Equations
- iv. Piecewise defined functions and Heaviside functions

- II. Systems of Equations
- a. Write system of two first order equations as second order ODE and vice versa.
 - b. Solve constant coefficient homogeneous systems of equations.
 - i. Use $\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$ to find phase portrait
 - ii. Fundamental, Principal Matrix Solutions
 - c. Phase Plane and Phase Portraits
 - i. Vector Fields
 - ii. Direction Fields
 - iii. Equilibrium Solutions
 - iv. y vs x and $x(t)$, $y(t)$ plots
 - d. Eigenvalue Problems
 - i. Solve for eigenvalues and eigenfunctions.
 - ii. Identify behavior of equilibria
 - iii. Use to construct solutions to linear systems
 - e. Applications – Mixture Problems, Block-Spring Systems, etc