

Differential Equations Review I

I. First Order Differential Equations

a. Separation of Variables

i. $\frac{dy}{dt} = f(t)g(y) \Rightarrow \int \frac{dy}{g(y)} = \int f(t) dt$

ii. General Solutions – Implicit and Explicit

iii. Initial Value Problems – Particular Solutions

b. Linear Differential Equations

i. Find integrating factors and solve initial value problems

ii. $y' + a(x)y = f(x)$

$$\mu(x) = \exp \int a(x) dx \Rightarrow (\mu y)' = \mu f$$

$$y(x) = \frac{1}{\mu(x)} \left[\int \mu(t)f(t) dt + C \right]$$

c. Exact Equations. $M(x, y) dx + N(x, y) dy = 0$.

i. Condition $M_y = N_x$

ii. Integrating factor, and finding solution $u(x, y) = C$, using

$$\frac{\partial u}{\partial x} = M, \quad \frac{\partial u}{\partial y} = N.$$

II. Second Order Differential Equations

a. Homogeneous, Constant Coefficient Equations $y'' + ay' + by = 0$

b. Solutions - $y(x) = e^{rx}, r^2 + ar + b = 0$.

i. Two, real distinct solutions $y = c_1 e^{r_1 x} + c_2 e^{r_2 x}$

ii. One real solution $y = (c_1 + c_2 x) e^{rx}$

iii. Two complex conjugate solutions $y = (c_1 \cos \beta x + c_2 \sin \beta x) e^{\alpha x}$

c. Theory

i. Existence-Uniqueness for Linear Initial Value Problems

ii. Linearly Independent/ Linearly Dependent

iii. Wronskian $W(y_1, y_2) = y_1 y_2' - y_1' y_2$

d. Method of Reduction of Order $y_2(x) = v(x)y_1(x)$

III. Applications

i. Free Fall

ii. Growth and Decay $y(t) = y_0 e^{kt}$

1. Populations

2. Radioactivity – Half Life

iii. Newton's Law of Cooling $T(t) = T_a + (T_0 - T_a)e^{-kt}$

iv. Orthogonal Curves $y_2' = -\frac{1}{y_1'}$

v. Simple Harmonic Motion, $x'' + \omega^2 x = 0$, $\omega = \sqrt{\frac{k}{m}}$

IV. Methods of Integration

a. Substitution

b. Integration by parts $\int u dv = uv - \int v du$

c. Trigonometric Integrals $\int \sin^n x dx$, $\int \cos^n x dx$,

d. Trigonometric Substitution

i. $x^2 + a^2$ - tangent substitution

ii. $a^2 - x^2$ - sine substitution

e. Integration using Partial Fraction Decomposition

V. Integrals you should be able to do (or similar ones)

$\int x^n dx$	$\int \frac{1}{x} dx$	$\int e^{ax} dx$	$\int a^x dx$
$\int \sin ax dx$	$\int \cos ax dx$	$\int \sec^2 ax dx$	$\int \csc^2 ax dx$
$\int \sec x \tan x dx$	$\int \csc x \cot x dx$	$\int \sinh ax dx$	$\int \cosh ax dx$
$\int \tan ax dx$	$\int \cot ax dx$	$\int \frac{1}{x^2 + a^2} dx$	$\int \frac{1}{\sqrt{a^2 - x^2}} dx$
$\int \sec ax dx$	$\int \ln x dx$	$\int x^n e^{ax} dx$	$\int \frac{1}{x^2 - a^2} dx$
$\int \sin^2 ax dx$	$\int \cos^2 ax dx$	$\int \sin ax \cos bxdx$	$\int \sin ax \sin bxdx$
$\int \frac{dx}{a + bx}$	$\int \frac{dx}{(x - a)(x - b)}$	$\int e^{ax} \cos bxdx$	$\int e^{ax} \sin bxdx$