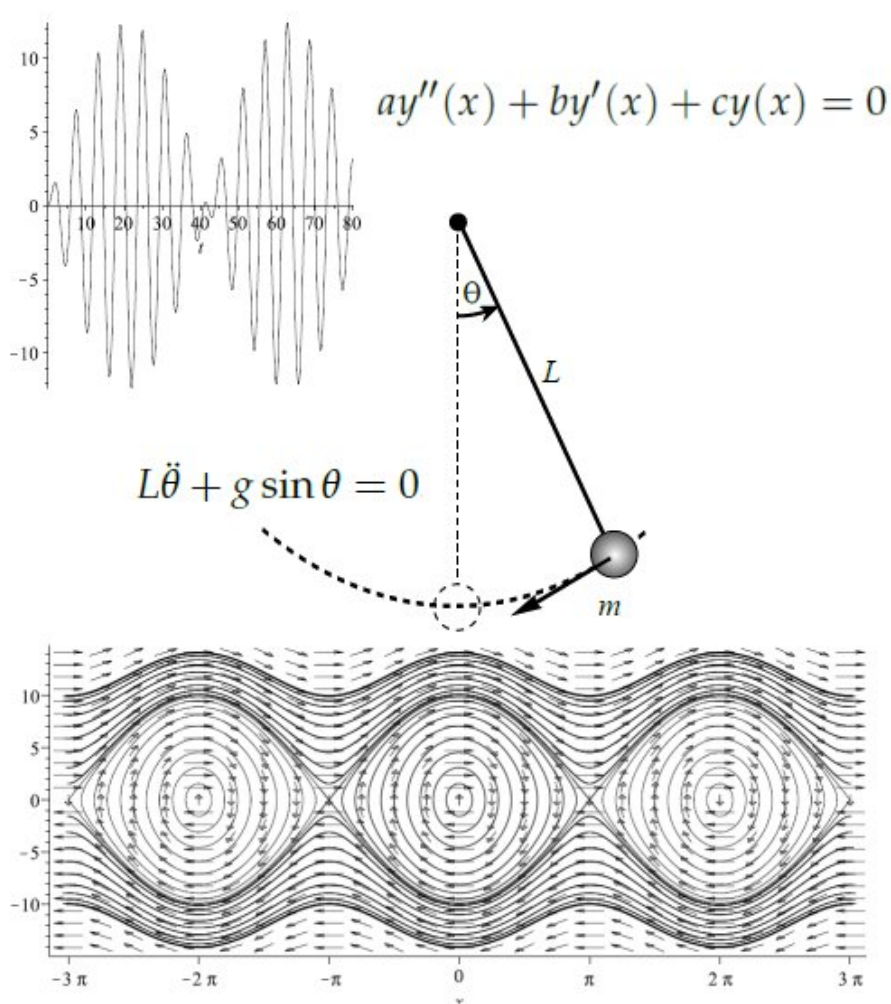


RUSSELL L. HERMAN

A FIRST COURSE
IN DIFFERENTIAL EQUATIONS
FOR SCIENTISTS AND ENGINEERS



R. L. HERMAN - VERSION DATE: AUGUST 17, 2018

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*Dedicated to those students who have endured
previous versions of my notes.*

