MAT 361 Exam III **Name** _____

Instructions:

- Place your name on all of the pages.
- Do all of your work in this booklet. Do not tear off any sheets.
- Show all of your steps in the problems for full credit.
- Be clear and neat in your work. Any illegible work, or scribbling in the margins, will not be graded.
- Put a box around your answers when appropriate..
- If you need more space, you may use the back of a page and write *On back of page #* in the problem space or the attached blank sheets. **No other scratch paper is allowed.**

Try to answer as many problems as possible. Provide as much information as possible. Show sufficient work or rationale for full credit. Remember that some problems may require less work than brute force methods.

If you are stuck, or running out of time, indicate as completely as possible, the methods and steps you would take to tackle the problem. Also, indicate any relevant information that you would use. Do not spend too much time on one problem. **Pace yourself.**

Pay attention to the point distribution. Not all problems have the same weight.

Page	Pts	Score
1	**20**	
2	**14**	
3	**11**	
4	**10**	
Total	**55**	

Bonus

i. Rewrite using Heaviside functions: $f(t)=\begin{cases} t, & 0\le t\le 3 \\ 0, & 3\le t \end{cases}$.

ii. Evaluate e^{iAt} for $A=\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$.

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1. (12 pts) Find the following Laplace transforms. [No integration is necessary!]
   a. $L[3e^{2t} + 2e^{-3t}] =$

   b. $L[e^{-2t} \sin 5t] =$

   c. $L[t \sinh t] =$

   d. $L\left[\int_0^t e^{2(u-t)} \cos 3u \, du\right] =$

2. (4 pts) Solve using the Laplace transform method: $y' + y = e^t$, $y(0) = 1$.

3. (4 pts) Use the Convolution Theorem to evaluate $L^{-1}\left[\frac{1}{s^2(s-1)}\right]$

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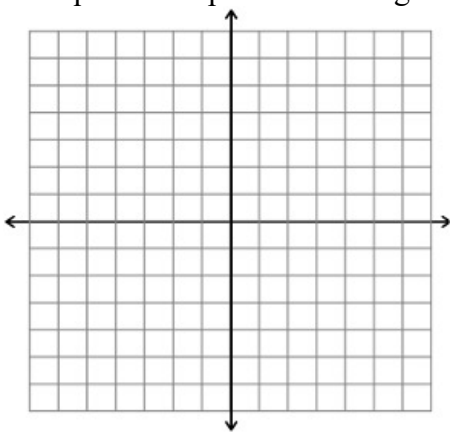
4. (5 pts) Consider the system $\begin{cases} x' = 4x \\ y' = -2y \end{cases}$. Using $\frac{dy}{dx}$, find the family of solution curves. Sketch these curves in each quadrant of the *xy*-phase plane. What type of equilibrium point is the origin? _____



5. (9 pts) Find the following inverse Laplace transforms.

   a. $L^{-1}\left[\frac{s}{(s-3)(s+2)}\right] =$

   b. $L^{-1}\left[\frac{se^{-\pi s}}{s^2+9}\right] =$

   c. $L^{-1}\left[\frac{s-3}{s^2-6s+18}\right] =$

6. (7 pts) A tank initially contains 100 liters of fresh water. Brine containing 25 grams per liter of salt flows into the tank at a rate of 4 liters per minute and the mixture, constantly stirred, runs out at the same rate.
   a. Write down the differential equation governing the rate of change of the amount of salt in the tank and the initial conditions.

   b. Solve the initial value problem.

   c. How long does it take for the quantity of salt in the tank to become 1 kilogram?

7. (4 pts) Use the Laplace transform method to solve the initial value problem: $y''-y'=2\delta(t-3),\ y(0)=1,\ y'(0)=0.$

8. (10 pts) Consider the system of differential equations: $\begin{cases} x' = -x + 6y \\ y' = x - 2y \end{cases}$.

   a. Convert the system to one second order differential equation for $x(t)$.

   b. Find the eigenvalues and corresponding eigenvectors of the coefficient matrix.

   c. Give the Fundamental Solution Matrix, Φ, and the general solution, $\mathbf{x}(t) = \begin{bmatrix} x(t) \\ y(t) \end{bmatrix}$ for this problem.

   d. What is the solution satisfying $\mathbf{x}(0) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$?

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Extra Space