MAT 361 Exam II Name ____________________________

Instructions:
1. Do all of your work on this sheet.
2. **Show all of your steps** in problems for full credit.
3. **Be clear and neat** in your work. Any illegible work, or scribbling in the margins, will not be graded.
4. Place your **answers in a box**.
5. If you need more space, you may use the back of the page and write **On back** in the problem space.
6. All answers should be exact!

1. (14 pts) Answer the following:

a. Identify the ordinary, regular singular, and irregular singular points in the equation:

$$xy''+\frac{2(x-2)}{x-1}y'+\frac{5x}{(x-1)^2}y=0.$$

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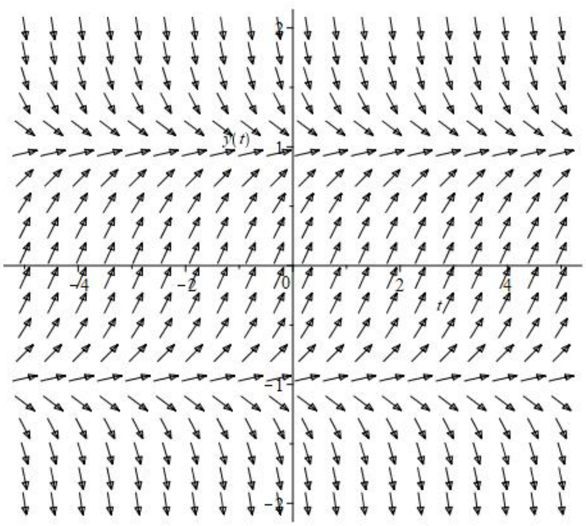
b. Sketch solution curves on the given direction field satisfying

$$y(0)=1.5,\ y(0)=-1.5,\ y(-2)=0.$$



c. Indicate the form you should guess for the particular solution:

i. $2y''+2y'+1=e^{-2x}$.

ii. $y''-3y'+2y=6e^{x}$.

iii. $y''+y'=2\cos x$

iv. $x^2y''+5xy'+4y=x^3$.

Column		Score
1	14 pts	
2	11 pts	
3	12 pts	
4	13 pts	
Total	50 pts	

2. (7 pts) Solve the differential equations for $x>0$:

a. $x^2y''+15xy'+48y=0,\quad y(1)=1, y'(1)=0.$

b. $x^2y''+3xy'+17y=0.$

3. (4 pts) Consider the problem $y'=\dfrac{y}{x+1},\ y(0)=2.$

a. Use Euler's Method to approximate $y(0.6)$ using a step size of $h=0.2$.

Bonus

b. Use the exact solution to find the numerical error in part a.

4. (12 pts) Consider the initial value problem: $y''+7y'+12y=2x$.

a. Find the general solution using the Method of Undetermined Coefficients.

b. Find a particular solution using the Method of Variation of Parameters.

c. For the above equation satisfying the initial conditions $y(0)=1, \quad y'(0)=0$, use the Taylor Series Method to approximate the solution of by a third degree polynomial in *x*.

6. (3 pts) Solve $x''+x=2\sin(t)$.

7. (10 pts) Consider the equation: $2xy''+y'+2y=0$.

a. Use the power series method to find the first three nonzero terms in one series solution.

b. When applying the Frobenius Method to this differential equation, we find

$$\sum_{n=0}^{\infty}\left[2(n+r)(n+r-1)+(n+r)\right]c_n z^{n+r-1}+2\sum_{n=0}^{\infty}c_n z^{n+r}=0.$$

i. What is the indicial equation?

ii. What are the roots of the indicial equation?

iii. Find the recursion relation for the smallest root.

Extra Space