MAT 361 Exam I **Name** _____

Instructions:
1. Do all of your work on this sheet.
2. **Show all of your steps** in problems for full credit.
3. **Be clear and neat** in your work. Any illegible work, or scribbling in the margins, will not be graded.
4. Place your **answers in a box**.
5. If you need more space, you may use the back of the page and write **On back** in the problem space.
6. All answers should be exact!

Column		Score
1	12 pts	
2	15 pts	
3	10 pts	
4	13 pts	
Total	50 pts	

1. (4 pts) Quickies – answer the following:

a. Give Euler's Formula:

$e^{i\theta} =$ _____

b. Write the solution for the Newton's Law of cooling for a body at temperature $T(t)$ placed in a room of temperature T_a.

2. (8 pts) Find the general solution to the given differential equation. When an initial condition is given, find the particular solution.

a. $\dfrac{dy}{dx} = x^4\sqrt{1-y^2}.$

b. $\dfrac{dy}{dx} - \dfrac{1}{x}y = x^2e^x, \ y(1) = 2.$

3. (12 pts) Find the general solution to the given differential equations. When an initial condition is given, find the particular solution.

a. $y'' - 4y' = 0,$ for $y = y(x).$

b. $16x'' + 8x' + x = 0, x(0) = 1, x'(0) = 0,$ for $x = x(t).$

c. $2y'' - 2y' + y = 0, \ y = y(x).$

4. (3 pts) Show that $y_1(x) = x$ and $y_2(x) = xe^x$ are linearly independent.

5. (5 pts) Determine if $(x + y\cos x)\,dx + \sin x\,dy = 0$ is exact, then find the solution.

6. (5 pts) Consider the differential equation
$$x^2y'' + xy' - 4y = 0,\ \ x > 0\,.$$
a. Verify that $y(x) = x^2$ is a solution.

b. Find a second linearly independent solution using the method of reduction of order.

7. (7 pts) The initial mass of an iodine isotope is 350g. The half-life of the isotope is 20 days.
a. Find the decay constant and write the equation for the mass as a function of time, $y(t)$.

b. Determine the iodine mass after 90 days.

8. (4 pts) A 1.50 kg mass oscillates on a spring with spring constant 7.50 N/m and damping constant 0.070 kg/s.
a. Write the differential equation for the position of the mass.

b. What type of damped oscillation is this?

9. (2 pts) A 10.0 kg ball is dropped from a cliff. The force of air resistance is seven times the speed. Write the governing differential equation for this free fall in terms of $v, \dfrac{dv}{dt}, g$.

Bonus: Find the terminal velocity for Problem 9.