MAT 361 **Name ______________________**

Instructions:

- Place your name on all of the pages.
- Do all of your work in this booklet. Do not tear off any sheets.
- Show all of your steps in the problems for full credit.
- Be clear and neat in your work. Any illegible work, or scribbling in the margins, will not be graded.
- Put a box around your answers when appropriate..
- If you need more space, you may use the back of a page and write *On back of page #* in the problem space or the attached blank sheets. **No scratch paper is allowed.**

Try to answer as many problems as possible. Provide as much information as possible. Show all work for full credit.

If you are stuck, or running out of time, indicate as completely as possible, the methods and steps you would take to tackle the problem. Also, indicate any relevant information that you would use. Do not spend too much time on one problem. **Pace yourself.**

Pay attention to the point distribution. Not all problems have the same weight.

Page	Pts	Score
1	21	
2	17	
3	13	
4	13	
5	11	
6	13	
7	12	
Total	100	

Bonus: A 400-gal tank initially contains 100 gal of brine containing 50 lb of salt. Brine containing 1 lb of salt per gallon enters the tank at a rate of 5 gal/s, and the well mixed brine in the tank flows out at a rate of 3 gal/s. How much salt will be the tank contain when it is full of brine?

Good Luck on your exam and all future endeavors!

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1. (6 pts) Find the Laplace transform of the following:

   a. $L[e^{2t-1}] =$

   b. $L[e^{-t}\cos 3t] =$

2. (10 pts) Find the inverse Laplace transform of the following:

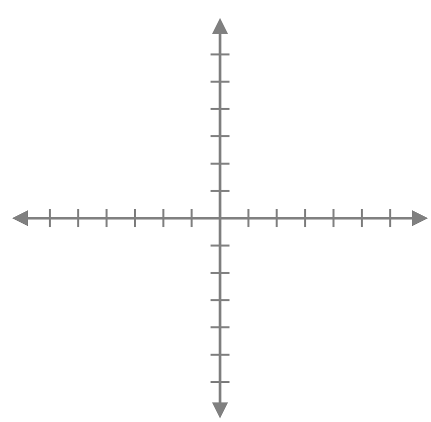
   a. $L^{-1}\left[\dfrac{s+2}{s^2-4}\right] =$

   b. $L^{-1}\left[\dfrac{s+3}{s^2+8s+17}\right] =$

   c. $L^{-1}\left[\dfrac{e^{-4s}}{s+7}\right] =$

   d. $L^{-1}\left[\dfrac{1}{s^2(s^2+4)}\right]$, using the Convolution Theorem

3. (5 pts) Find and classify the equilibrium solutions of the equation $\dfrac{dy}{dt} = 2(y+4)(y-3)$. Sketch several solutions in the *ty*-plane supporting your answer.



4. (17 pts) Solve the differential equations below. If an initial condition is given, find the particular solution; otherwise give the general solution.

   a. $y' + y^2 \sin x = 0,\ y(0) = 1/2.$

   b. $\dfrac{dy}{dx} = \dfrac{x^3 - 2y}{x}.$

   c. $x^2 y'' - 3xy' + 13y = 0,\ \ x > 0.$

   d. $y'' - y' - 6y = 20e^{-2t},\ y(0) = 0,\ y'(0) = 6.$

   e. $(2xy + y^3)\,dx + (x^2 + 3xy^2 - 2y)\,dy = 0.$

5. (5 pts) Use the Method of Variation of Parameters to obtain the general solution: $y''+y=\sin t$.

6. (4 pts) Find the first three nonvanishing terms in a series expansion about $x=0$ the initial value problem $y''=\sin(y),\ y(0)=0, y'(0)=1$ using the Taylor series method.

7. (4 pts) An instrument at an initial temperature of 40 ºC is placed in a room whose temperature is 20 ºC. For the next 5 hours the room temperature gradually rises and is given by $T_0(t)=20+10t,$ where *t* is measured in hours. Use Newton's Law of Cooling $\dfrac{dT}{dt}=-k(T-T_0)$ with $k=1$, to determine the temperature of the instrument $T(t)$ at any time $t>0$. At what time will the instrument temperature be the same as the room temperature?

8. (3 pts) For the given points, determine if they are ordinary, regular singular or irregular singular points of the differential equation: $x^2(x-2)y''-(x-1)y'+xy=0$.
   a. $x=0$.

   b. $x=1$.

   c. $x=2$.

9. (5 pts) Solve using the Laplace transform method:
   $y''-5y'+6y=0,\ y(0)=2,\ y'(0)=1$.

10. (5 pts) Find the recursion relation and give the general power series solution to five terms of differential equation $y''-2xy'+2y=0$.

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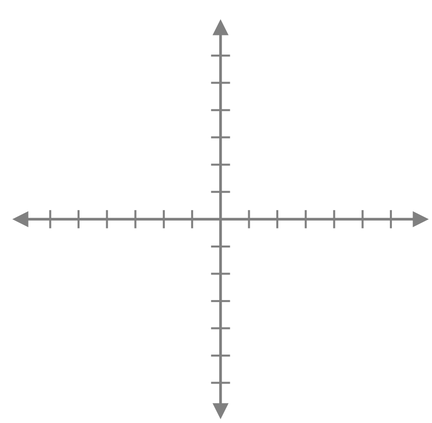
11. (4 pts) A mass of 10 kg is attached to a long spring with spring constant 1000 N/m. It is initially displaced one meter from equilibrium and released. The system is subject to damping characterized by a damping constant of 200 kg/s and an external force of $F = 20\sin 10t$ N. The governing differential equation is $10\ddot{x} + 200\dot{x} + 1000x = 20\sin 10t$.

    a. Solve this initial value problem.

    b. How would you classify the unforced motion?

12. (3 pts) One solution of $t^2x'' - 4tx' + 6x = 0$ is $x_1(t) = t^2$. Use the Method of Reduction of Order to find a second solution.

13. (4 pts) Construct a bifurcation diagram for $\frac{dy}{dt} = \mu y - y^3$.



14. (8 pts) Consider the system of differential equations: $\begin{cases} x' = x + y \\ y' = 3x - y \end{cases}$.

a. Convert the system to one second order differential equation.

b. Determine the eigenvalues and eigenfunctions of the coefficient matrix.

c. Use your answer in part b. to write the general solution.

15. (5 pts) Find the equilibrium solutions of the system: $\begin{cases} x' = -x - 2x^2 + xy \\ y' = -y + 7xy - 2y^2 \end{cases}$. Classify any points with $x, y > 0$.

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16. (12 pts) Describe whatever you can about the following plots by sketching solutions, discussing eigenvalues, and classifying all equilibrium points.